

TOTALLY UNIMODULAR  
DIRECTED HYPERGRAPHS & APPLICATIONS

*Linear Algebra and Its Applications*, Vol 230, p101, November 1995

Collette R. Coullard and Peh H. Ng

A Leontief directed hypergraph is a generalization of a directed graph, where arcs have multiple (or no) tails and at most one head. We define a class of Leontief directed hypergraphs via a forbidden structure called an odd pseudocycle. We show that the vertex-hyperarc incidence matrices of the hypergraphs in this class are totally unimodular. Indeed, we show that this is the largest class with that property. We define two natural subclasses of this class (one obtained by forbidding pseudocycles and the other obtained by forbidding pseudocycles and the so-called doublecycles) and we describe some structural properties of the bases and circuits of the members of these classes. We present examples of Leontief directed hypergraphs that are graphic, cographic and neither graphic nor cographic.