

Questions

1. Using a sign chart, find the values of x that satisfy $\frac{x^3 - x^2 - 8x + 12}{3 - x} > 0$. Express your solution in both set notation and interval notation.
2. Using a sign chart, find the values of x that satisfy $\frac{(5x - 6)|2x - 4|}{x} > 0$. Express your solution in both set notation and interval notation.
3. Using a sign chart, find the values of x that satisfy $\frac{(4x + 7)\sqrt{1 - x}}{x + 10} \leq 0$. Express your solution in both set notation and interval notation.

Solutions

1. Using a sign chart, find the values of x that satisfy $\frac{x^3 - x^2 - 8x + 12}{3 - x} > 0$. Express your solution in both set notation and interval notation.

$$f(x) = \frac{x^3 - x^2 - 8x + 12}{3 - x}$$

Factor numerator:

factors of $a_0 = 12: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$a_3 = 1: \pm 1$

\Rightarrow potential rational zeros are

start checking: ± 1 won't work by inspection
(I can look at it and see $f(\pm 1) \neq 0$)

$$f(2) = (2)^3 - (2)^2 - 8(2) + 12 = 0 \checkmark$$

$\Rightarrow x - 2$ will factor.

$$\begin{array}{r} x^2 + x - 6 \\ x-2 \overline{) x^3 - x^2 - 8x + 12} \\ \underline{x^3 - 2x^2} \\ x^2 - 8x + 12 \\ \underline{x^2 - 2x} \\ -6x + 12 \\ \underline{-6x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \text{so } x^3 - x^2 - 8x + 12 &= (x-2)(x^2 + x - 6) \\ &= (x-2)(x-2)(x+3) \\ &= (x-2)^2(x+3) \end{aligned}$$

$$\text{so } f(x) = \frac{(x-2)^2(x+3)}{3-x}$$

Let's do sign chart using end behaviour and multiplicity of roots:

Zeros: $x=2$, even mult., f does not change sign.

$x=-3$, odd mult., f changes sign.

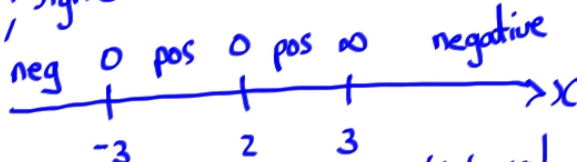
vertical asymptote: $x=3$, odd mult., f changes sign.

End behaviour:

$$f(x) \sim \frac{(x)^2(x)}{(-x)} \sim -x^2$$

$\Rightarrow f(x) < 0$ as $x \rightarrow \infty$

Sign chart:



So $f(x) > 0$ when

$$\begin{aligned} &-3 < x < 2 \text{ or } 2 < x < 3 \text{ (interval notation)} \\ &x \in (-3, 2) \cup (2, 3) \text{ (set notation)} \end{aligned}$$

2. Using a sign chart, find the values of x that satisfy $\frac{(5x-6)|2x-4|}{x} > 0$. Express your solution in both set notation and interval notation.

$$f(x) = \frac{(5x-6)|2x-4|}{x} \quad \text{already factored!}$$

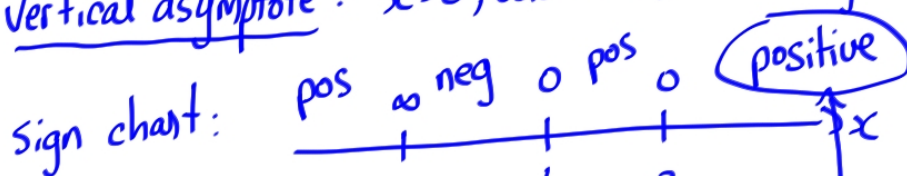
Note absolute value.

Sign chart from zeros/multiplicity:

Zeros: $x = \frac{6}{5}$, odd mult. \Rightarrow f changes sign.

$x = 2$, absolute value \Rightarrow f does not change sign.

Vertical asymptote: $x = 0$, odd mult, \Rightarrow f changes sign.



$f(x) > 0$ when:

$$x < 0 \text{ or } \frac{6}{5} < x < 2 \text{ or } x > 2.$$

whoops! Need end behaviour to continue.

$$f(x) \sim \frac{(5x)|2x|}{x} = 10x \text{ if } x \rightarrow \infty.$$

So $f(x) > 0$ if $x \rightarrow \infty$.

$$x \in (-\infty, 0) \cup \left(\frac{6}{5}, 2\right) \cup (2, \infty)$$

3. Using a sign chart, find the values of x that satisfy $\frac{(4x+7)\sqrt{1-x}}{x+10} \leq 0$. Express your solution in both set notation and interval notation.

$$f(x) = \frac{(4x+7)\sqrt{1-x}}{x+10} \quad \text{factored!}$$

Notice $\sqrt{\quad}$.

Zeros: $x = -7/4$, f changes sign (odd mult).

$x = 1$. Square Root, so we need to look at domain.

$$1-x \geq 0$$

$$x \leq 1$$

$\Rightarrow f$ is not defined for $x > 1$.

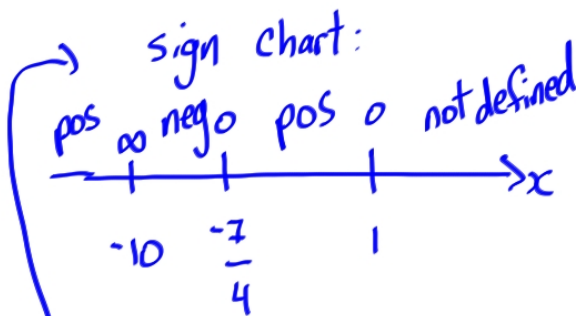
Vertical Asymptotes: $x = -10$, f changes sign (odd mult.)

End behaviour (have to look at $x \rightarrow -\infty$)

$$f(x) \sim \frac{(4x)\sqrt{-x}}{x} \sim 4\sqrt{-x}$$

which is positive as $x \rightarrow -\infty$.

(Note that you could always switch to looking at signs of factors in intervals if end behaviour causes you some difficulty)



So $f(x) \leq 0$ when

$$-10 < x \leq -7/4$$

or

$$x \in (-10, -7/4]$$