Questions

1. Using a sign chart, find the values of x that satisfy $\dfrac{x^3 - x^2 - 8x + 12}{3 - x} > 0$. Express your solution in both set notation and interval notation.

2. Using a sign chart, find the values of x that satisfy $\dfrac{(5x-6)|2x-4|}{x} > 0$. Express your solution in both set notation and interval notation.

3. Using a sign chart, find the values of x that satisfy $\dfrac{(4x+7)\sqrt{1-x}}{x+10} \leq 0$. Express your solution in both set notation and interval notation.

Solutions

1. Using a sign chart, find the values of x that satisfy $\dfrac{x^3 - x^2 - 8x + 12}{3 - x} > 0$. Express your solution in both set notation and interval notation.

$$f(x) = \frac{x^3 - x^2 - 8x + 12}{3 - x}$$

Factor numerator:

factors of $a_0 = 12$: $\underbrace{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$

$a_3 = 1$: ± 1

$\Rightarrow$ potential rational zeros are

start checking: ± 1 won't work by inspection

(I can look at at it and see $f(\pm 1) \neq 0$)

$f(2) = (2)^3 - (2)^2 - 8(2) + 12 = 0$ ✓

$\Rightarrow$ $x - 2$ will factor.

$$\begin{array}{r} x^2 + x - 6 \\ x - 2 \overline{\smash{)}\, x^3 - x^2 - 8x + 12} \\ \underline{x^3 - 2x^2} \\ x^2 - 8x + 12 \\ \underline{x^2 - 2x} \\ -6x + 12 \\ \underline{-6x + 12} \\ 0 \end{array}$$

so $x^3 - x^2 - 8x + 12 = (x-2)(x^2 + x - 6)$

$= (x-2)(x-2)(x+3)$

$= (x-2)^2(x+3)$

So $f(x) = \dfrac{(x-2)^2(x+3)}{3-x}$

End behaviour:

$f(x) \sim \dfrac{(x)^2(x)}{(-x)} \sim -x^2$

$\Rightarrow f(x) < 0$ as $x \to \infty$

Let's do sign chart using end behaviour and multiplicity of roots:

Zeros: $x = 2$, even mult., f does not change sign.

$x = -3$, odd mult., f changes sign.

vertical asymptote: $x = 3$, odd mult. f changes sign.

Sign chart:

neg	-3	pos	2	pos	3	negative
	0		0		∞	

So $f(x) > 0$ when $-3 < x < 2$ or $2 < x < 3$ (interval notation)

$x \in (-3, 2) \cup (2, 3)$ (set notation)

2. Using a sign chart, find the values of x that satisfy $\dfrac{(5x-6)|2x-4|}{x} > 0$. Express your solution in both set notation and interval notation.

$f(x) = \dfrac{(5x-6)\,|2x-4|}{x}$ already factored! Note absolute value.

Sign chart from zeros/multiplicity:

<u>Zeros</u>: $x = 6/5$, odd mult. $\Rightarrow$ f changes sign.

$x = 2$, absolute value $\Rightarrow$ f does not change sign.

<u>Vertical asymptote</u>: $x=0$, odd mult. $\Rightarrow$ f changes sign.

Sign chart: pos ∞ neg 0 pos 0 positive

$0 \qquad \frac{6}{5} \qquad 2 \qquad x$

Whoops! Need end behaviour to continue.

$f(x) \sim \dfrac{(5x)\,|2x|}{x} = 10x$ if $x \to \infty$.

So $f(x) > 0$ if $x \to \infty$.

$f(x) > 0$ when:

$x < 0$ or $\frac{6}{5} < x < 2$ or $x > 2$.

$x \in (-\infty, 0) \cup \left(\frac{6}{5}, 2\right) \cup (2, \infty)$

3. Using a sign chart, find the values of x that satisfy $\dfrac{(4x+7)\sqrt{1-x}}{x+10} \leq 0$. Express your solution in both set notation and interval notation.

$f(x) = \dfrac{(4x+7)\sqrt{1-x}}{x+10}$ factored. Notice $\sqrt{\ }$.

<u>ZEROS</u>: $x = -7/4$, f changes sign (odd mult).

$x = 1$. Square Root, so we need to look at domain.

$1 - x \geq 0$

$x \leq 1$

$\Rightarrow f$ is not defined for $x > 1$.

<u>Vertical Asymptotes</u>: $x = -10$, f changes sign (odd mult)

<u>End behaviour</u> (have to look at $x \to -\infty$)

$f(x) \sim \dfrac{(4x)\sqrt{-x}}{x} \sim 4\sqrt{-x}$

which is positive as $x \to -\infty$.

(Note that you could always switch to looking at signs of factors in intervals if end behaviour causes you some difficulty)

Sign chart:

pos ∞ neg 0 pos 0 not defined

x: -10, $-\frac{7}{4}$, 1

So $f(x) \leq 0$ when

$-10 < x \leq -7/4$

or

$x \in \left(-10, -7/4\right]$