Questions

1. Solve $\frac{8}{x} + \frac{2}{5} = -\frac{2}{x}$.

2. Solve $\frac{x+1}{2x} = \frac{2}{3}$.

3. Solve $\frac{2}{2x+5} = \frac{4}{x-4}$.

4. Solve $\frac{3}{x+5} = \frac{3}{3x-2}$.

5. Solve $7 - \frac{x}{x+5} = \frac{5}{x+5}$.

6. Solve $\frac{8x}{4x^2-1} = \frac{3}{2x+1} + \frac{3}{2x-1}$.

7. Solve $\frac{6}{x-5} + \frac{3x+1}{x^2-2x-15} = \frac{5}{x+3}$.

8. Solve $\frac{6}{x-3} = \frac{-5}{x-2} - \frac{5}{x^2-5x+6}$.

9. Solve $\frac{x+11}{x^2-5x+4} + \frac{3}{x-1} = \frac{5}{x-4}$.

10. It takes Rhonda 150 minutes to clean the house. It takes Pauline 100 minutes to clean the house. Working together, how long will it take them to clean the house?

Solutions

1. Lowest common denominator is $5x$.

$$\left(\frac{8}{x}\right)5x + \left(\frac{2}{5}\right)5x = \left(-\frac{2}{x}\right)5x$$
$$40 + 2x = -10$$
$$2x = -10 - 40$$
$$x = -\frac{50}{2} = -25$$

Check:

$$\frac{8}{(-25)} + \frac{2}{5} = -\frac{2}{(-25)}$$
$$-\frac{8}{25} + \frac{10}{25} = \frac{2}{25}$$
$$\frac{2}{25} = \frac{2}{25} \text{ it's a solution}$$

2. LCD is $6x$.

$$\left(\frac{x+1}{2x}\right)6x = \left(\frac{2}{3}\right)6x$$
$$3x + 3 = 4x$$
$$3 = 4x - 3x$$
$$3 = x$$

Check:

$$\frac{(3)+1}{2(3)} = \frac{2}{3}$$
$$\frac{4}{6} = \frac{2}{3}$$
$$\frac{2}{3} = \frac{2}{3} \text{ it's a solution!}$$

3. LCD is $(2x+5)(x-4)$.

$$\left(\frac{2}{\cancel{2x+5}}\right)\cancel{(2x+5)}(x-4) = \left(\frac{4}{\cancel{x-4}}\right)(2x+5)\cancel{(x-4)}$$
$$2(x-4) = 4(2x+5)$$
$$2x-8 = 8x+20$$
$$-6x = 28$$
$$x = \frac{28}{-6} = -\frac{14}{3}$$

Check:

$$\frac{2}{2(-14/3)+5} = \frac{4}{(-14/3)-4}$$
$$\frac{2}{-28/3+15/3} = \frac{4}{(-14/6)-12/3}$$
$$\frac{2}{-13/3} = \frac{4}{-26/3}$$
$$\frac{6}{-13} = \frac{12}{-26}$$
$$\frac{6}{-13} = \frac{6}{-13}$$ it's a solution

4. LCD is $(x+5)(3x-2)$.

$$\left(\frac{3}{\cancel{x+5}}\right)\cancel{(x+5)}(3x-2) = \left(\frac{3}{\cancel{3x-2}}\right)(x+5)\cancel{(3x-2)}$$
$$3(3x-2) = 3(x+5)$$
$$3x-2 = \frac{\cancel{3}}{\cancel{3}}(x+5)$$
$$3x-2 = x+5$$
$$2x = 7$$
$$x = \frac{7}{2}$$

Check:

$$\frac{3}{(7/2)+5} = \frac{3}{3(7/2)-2}$$
$$\frac{3}{7/2+10/2} = \frac{3}{21/2-4/2}$$
$$\frac{3}{17/2} = \frac{3}{17/2}$$ it's a solution

5. LCD is $x+5$.

$$(7)(x+5) - \left(\frac{x}{\cancel{x+5}}\right)\cancel{(x+5)} = \left(\frac{5}{\cancel{x+5}}\right)\cancel{(x+5)}$$
$$7x+35-x = 5$$
$$6x = -30$$
$x = -5$ which is an extraneous solution, since it causes division by zero in original equation.

6. Factor polynomials.

$4x^2 - 1 = (2x - 1)(2x + 1)$ difference of squares

Looking at the equation, we now see the LCD is $(2x - 1)(2x + 1)$.

$$\left(\frac{8x}{\cancel{(2x-1)}\cancel{(2x+1)}}\right)\cancel{(2x-1)}\cancel{(2x+1)} = \left(\frac{3}{\cancel{2x+1}}\right)(2x-1)\cancel{(2x+1)} + \left(\frac{3}{\cancel{2x-1}}\right)\cancel{(2x-1)}(2x+1)$$

$$\begin{aligned} 8x &= 3(2x-1) + 3(2x+1) \\ 8x &= 6x - 3 + 6x + 3 \\ 8x &= 12x \\ -4x &= 0 \\ x &= \frac{0}{-4} = 0 \end{aligned}$$

Check:

$$\begin{aligned} \frac{8(0)}{4(0)^2 - 1} &= \frac{3}{2(0)+1} + \frac{3}{2(0)-1} \\ 0 &= 3 - 3 \text{ it's a solution} \end{aligned}$$

7. Factor polynomials.

$x^2 - 2x - 15 = (x + 3)(x - 5)$ Need two numbers whose product is -15 sum is -2: $3, -5$

Looking at the equation, we now see the LCD is $(x + 3)(x - 5)$.

$$\left(\frac{6}{\cancel{x-5}}\right)(x+3)\cancel{(x-5)} + \left(\frac{3x+1}{\cancel{(x+3)}\cancel{(x-5)}}\right)\cancel{(x+3)}\cancel{(x-5)} = \left(\frac{5}{\cancel{x+3}}\right)\cancel{(x+3)}(x-5)$$

$$\begin{aligned} 6(x+3) + 3x + 1 &= 5(x-5) \\ 6x + 18 + 3x + 1 &= 5x - 25 \\ 4x &= -44 \\ x &= -11 \end{aligned}$$

Check:

$$\begin{aligned} \frac{6}{(-11)-5} + \frac{3(-11)+1}{(-11)^2 - 2(-11) - 15} &= \frac{5}{(-11)+3} \\ \frac{6}{-16} + \frac{-32}{128} &= \frac{5}{-8} \\ \frac{3}{-8} + \frac{-1}{4} &= \frac{5}{-8} \\ -\frac{3}{8} - \frac{2}{8} &= -\frac{5}{8} \\ -\frac{5}{8} &= -\frac{5}{8} \text{ it's a solution} \end{aligned}$$

8. Factor polynomials.

$x^2 - 5x + 6 = (x - 3)(x - 2)$ Need two numbers whose product is 6 sum is -5: $-2, -3$

Looking at the equation, we now see the LCD is $(x-3)(x-2)$.

$$\frac{6}{x-3} = \frac{-5}{x-2} - \frac{5}{(x-3)(x-2)}$$

$$\left(\frac{6}{\cancel{x-3}}\right)\cancel{(x-3)}(x-2) = \left(\frac{-5}{\cancel{x-2}}\right)(x-3)\cancel{(x-2)} - \left(\frac{5}{\cancel{(x-3)(x-2)}}\right)\cancel{(x-3)(x-2)}$$

$$6(x-2) = -5(x-3) - 5$$
$$6x - 12 = -5x + 15 - 5$$
$$11x = 22$$
$$x = 2$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x = 2$ is not a solution. Therefore, the original equation has no solution.

9. Solve $\dfrac{x+11}{x^2-5x+4} + \dfrac{3}{x-1} = \dfrac{5}{x-4}$.

Factor $x^2 - 5x + 4$: Need two numbers whose product is 4 and sum is -5: $-4, -1$.

$x^2 - 5x + 4 = (x-4)(x-1)$.

The LCD for the the equation is $(x-4)(x-1)$. Multiply all terms in the equation by this LCD:

$$\frac{x+11}{x^2-5x+4} + \frac{3}{x-1} = \frac{5}{x-4}$$

$$\frac{x+11}{\cancel{(x-4)(x-1)}} \cdot \cancel{(x-4)(x-1)} + \frac{3}{\cancel{x-1}} \cdot (x-4)\cancel{(x-1)} = \frac{5}{\cancel{x-4}} \cdot \cancel{(x-4)}(x-1)$$

$$x + 11 + 3(x-4) = 5(x-1)$$
$$x + 11 + 3x - 12 = 5x - 5$$
$$4x - 1 = 5x - 5$$
$$4x - 5x = -5 + 1$$
$$-x = -4$$
$$x = 4$$

We aren't done until we verify this is actually a solution. Since $x = 4$ makes the LCD zero, this is <u>not</u> a solution since it would result in division by zero.

Therefore, $x = 4$ is an extraneous solution (meaning it is not a solution), and the original equation has no solution.

10. It takes Rhonda 150 minutes to clean the house. It takes Pauline 100 minutes to clean the house. Working together, how long will it take them to clean the house?

In 1 minute Rhonda will finish $\dfrac{1}{150}$ of the cleaning.
In 1 minute Pauline will finish $\dfrac{1}{100}$ of the cleaning.
Let x be the time it takes them to complete the cleaning together.

In 1 minute together they will finish $\frac{1}{x}$ of the cleaning.

$$\begin{aligned}\frac{1}{150}+\frac{1}{100}&=\frac{1}{x}\\ \frac{2}{300}+\frac{3}{300}&=\frac{1}{x}\\ \frac{5}{300}&=\frac{1}{x}\\ x&=60\end{aligned}$$

It will take 60 minutes to clean the house if they work together.