Questions

1. Sketch by hand $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and $(x-1)^2 + y^2 = 4$ on the same set of axes, and use the sketches to solve the system of equations

$$\begin{aligned} &\frac{x^2}{9} + \frac{y^2}{16} = 1 \\ &(x-1)^2 + y^2 = 4 \end{aligned}$$

Then check your solution to the system of equations by solving the system algebraically.

2. Sketch $36(x-2)^2 + 4(y+5)^2 = 144$.

3. Show $9x^2 + 4y^2 - 18x + 8y - 23 = 0$ is an ellipse by completing the square. Then sketch the ellipse.

4. Sketch $25y^2 - 9x^2 - 50y - 54x - 281 = 0$.

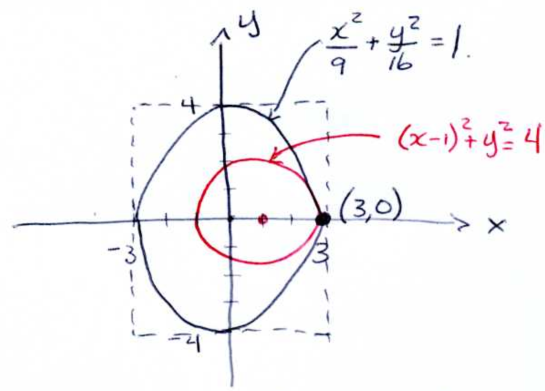
Solutions

1.

sketch $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

This is an ellipse, centered at $(0,0)$. $\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1$.

"spread" in x is $a=3$.
"spread" in y is $b=4$.



sketch $(x-1)^2+y^2=4=2^2$

This is a circle, centered at $(1,0)$ with radius 2.

It's a bit hard to tell when drawn by hand, but it looks like the only intersection is $(3,0)$.

Verify solution to $\left.\begin{array}{l}\frac{x^2}{9}+\frac{y^2}{16}=1 \\ (x-1)^2+y^2=4\end{array}\right\}$ is $\begin{array}{l}x=3\\ y=0\end{array}$:

From second equation, $y^2 = 4-(x-1)^2$. Substitute this into 1st equation:

$$\frac{x^2}{9} + \frac{(4-(x-1)^2)}{16} = 1$$ Now solve for x.

$16x^2 + 4 - (x-1)^2 = 144$

$16x^2 - x^2 + 2x - 1 = 140$

$15x^2 + 2x = 141$

$15x^2+2x-141=0$ use quadratic formula:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(15)(-141)}}{2(15)}$$

$x=3$ or $x = -47/15$

If $x=3$, $y^2 = 4-(3-1)^2 = 0$

So $(3,0)$ is a solution.

If $x = -\frac{47}{15}$, $y^2 = 4 - \left(-\frac{47}{15}-1\right)^2$

$= 4 - \left(\frac{62}{15}\right)^2$

$y^2 = \frac{-2944}{225} < 0$!

No real y value.

So only real valued solution is $x=3, y=0$.

2. Sketch $36(x-2)^2 + 4(y+5)^2 = 144$.

The addition and squaring make us think this is an ellipse.
Get it in standard form.

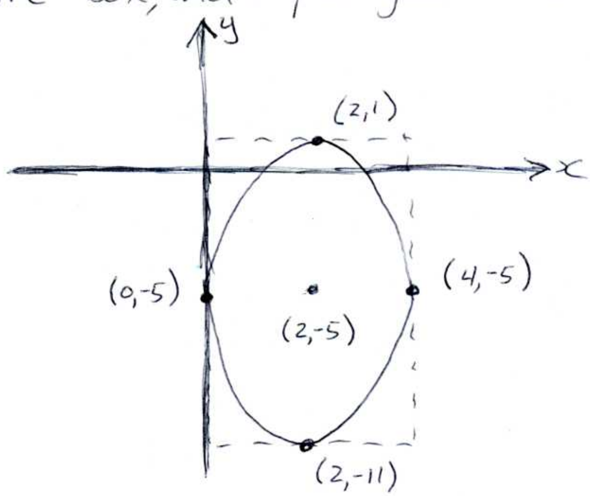
$$\left[36(x-2)^2 + 4(y+5)^2 = 144\right] \frac{1}{144}$$

$$\frac{(x-2)^2}{4} + \frac{(y+5)^2}{36} = 1$$

$$\frac{(x-2)^2}{2^2} + \frac{(y+5)^2}{6^2} = 1$$

center : $(2,-5)$

"spread" in x is $a=2$. (technically this is called the <u>semiminor axis</u>)
"spread" in y is $b=6$. (technically this is called the <u>semimajor axis</u>)
Draw the box, and ellipse goes in the box:



Add the axes at end, using the box to figure out where they should go.

3. Show $9x^2 + 4y^2 - 18x + 8y - 23 = 0$ is an ellipse by completing the square. Then sketch the ellipse.

You must complete the square in x and y:

$$
\begin{aligned}
9x^2 - 18x &= 9(x^2 - 2x) \\
&= 9(\underbrace{x^2 - 2x + 1} - 1) \\
&= 9([x-1]^2 - 1) \\
&= 9[x-1]^2 - 9
\end{aligned}
$$

$$
\begin{aligned}
4y^2 + 8y &= 4(y^2 + 2y) \\
&= 4(\underbrace{y^2 + 2y + 1} - 1) \\
&= 4([y+1]^2 - 1) \\
&= 4[y+1]^2 - 4.
\end{aligned}
$$

so $9x^2 + 4y^2 - 18x + 8y - 23$

$$
\begin{aligned}
&= (9x^2 - 18x) + (4y^2 + 8y) - 23 \\
&= 9(x-1)^2 - 9 + 4(y+1)^2 - 4 - 23 \\
&= 9(x-1)^2 + 4(y+1)^2 - 36 = 0
\end{aligned}
$$

replace with what we found above.

$$\Rightarrow \quad \frac{1}{36}\left[9(x-1)^2 + 4(y+1)^2 = 36\right]$$

$$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1 \quad \text{or} \quad \frac{(x-1)^2}{2^2} + \frac{(y+1)^2}{3^2} = 1.$$

This is an ellipse!

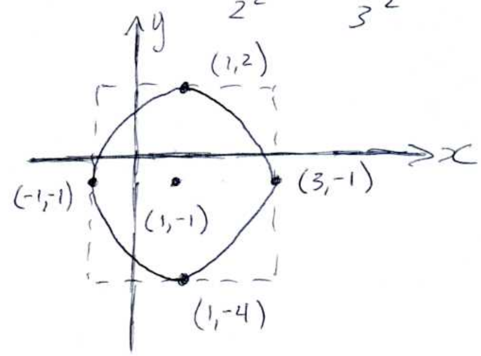
Center: $(1,-1)$

"spread" in x is $a=2$.

"spread" in y is $b=3$.

Draw the box

Add axes at end.



4. Sketch $25y^2 - 9x^2 - 50y - 54x - 281 = 0$.

First, complete the square in x and y.

$$25y^2 - 50y = 25(y^2 - 2y) = 25(\underbrace{y^2 - 2y + 1}-1) = 25((y-1)^2 - 1) = 25(y-1)^2 - 25$$

$$-9x^2 - 54x = -9(x^2 + 6x) = -9(\underbrace{x^2 + 6x + 9}-9) = -9((x+3)^2 - 9) = -9(x+3)^2 + 81$$

Our equation:

$$25y^2 - 9x^2 - 50y - 54x - 281 = \underbrace{25y^2 - 50y}_{\text{replace}} \underbrace{- 9x^2 - 54x}_{\text{replace}} - 281$$

use what we have from completing the square!

$$= 25(y-1)^2 - 25 - 9(x+3)^2 + 81 - 281$$

$$= 25(y-1)^2 - 9(x+3)^2 - 225 = 0$$

this is an hyperbola!

So get in better form!

$$\frac{1}{225}\left[25(y-1)^2 - 9(x+3)^2 = 225\right]$$

$$\frac{(y-1)^2}{9} - \frac{(x+3)^2}{25} = 1 \Rightarrow \frac{(y-1)^2}{3^2} - \frac{(x+3)^2}{5^2} = 1$$

Get Box:

Center $(-3, 1)$.

"spread" in x is $a = 5$.

"spread" in y is $b = 3$.

Lines through corners of box will be slant asymptotes.

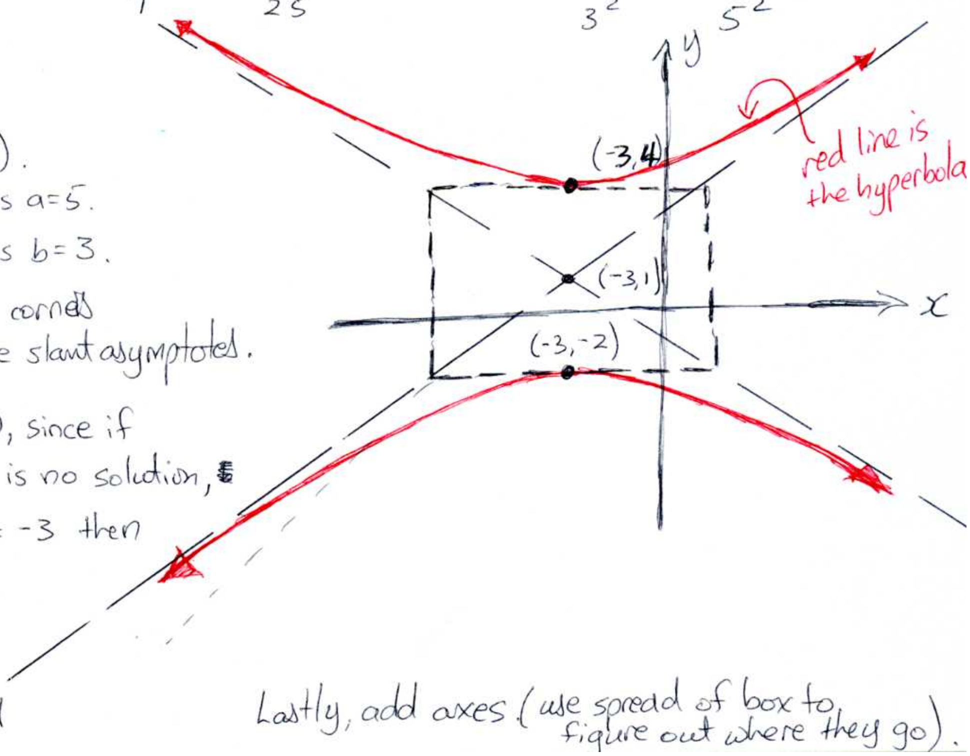
This opens up, since if $y = 1$, there is no solution.

However, if $x = -3$ then

$$\frac{(y-1)^2}{3^2} - 0 = 1$$

$$\Rightarrow y - 1 = \pm 3$$

$$y = -2, +4$$



Lastly, add axes (use spread of box to figure out where they go).