Questions

Include complex solutions in your answers.

1. Solve $(x+9)^2 = 21$.

2. Solve $(4x-3)^2 = 36$.

3. Solve $(5x-2)^2 - 25 = 0$.

4. Solve by completing the square $x^2 + 6x + 2 = 0$.

5. Solve by completing the square $x^2 - 14x = -48$.

6. Solve by completing the square $\dfrac{x^2}{3} - \dfrac{x}{3} = 3$.

7. Solve by completing the square $2y^2 - y = 15$.

8. Solve $x^2 - 2x = -7$.

9. Solve $3x^2 + 8x + 3 = 2$.

10. Sketch $f(x) = 2x^2 + 2x - 4$.

11. Sketch $w(x) = x^2 - 6x + 8$.

12. Sketch $g(x) = x^2 - 2x - 8$.

13. Sketch $r(x) = -3x^2 + 6x - 4$.

Solutions

1. Use the square root property, $w^2 = a \Rightarrow w = \pm\sqrt{a}$.

$$\begin{aligned}(x+9)^2 &= 21\\ x+9 &= \pm\sqrt{21}\\ x &= -9 \pm \sqrt{21}\end{aligned}$$

2. Use the square root property.

$$\begin{aligned}(4x-3)^2 &= 36\\ 4x-3 &= \pm\sqrt{36}\\ 4x &= 3 \pm 6\\ x &= \frac{3\pm 6}{4} = \frac{3+6}{4} \text{ or } \frac{3-6}{4}\\ x &= \frac{3+6}{4} \text{ or } \frac{3-6}{4}\\ x &= \frac{9}{4} \text{ or } -\frac{3}{4}\end{aligned}$$

3. Use the square root property.

$$(5x-2)^2-25=0$$
$$(5x-2)^2=25$$
$$5x-2=\pm\sqrt{25}$$
$$x=\frac{2\pm5}{5}$$
$$x=\frac{2+5}{5} \text{ or } \frac{2-5}{5}$$
$$x=\frac{7}{5} \text{ or } -\frac{3}{5}$$

4.

$x^2+6x+2=0$ To complete the square: $\left(\frac{6}{2}\right)^2=9$.

$x^2+6x+9-9+2=0$ underlined piece is a perfect square

$$(x+3)^2-7=0$$
$$(x+3)^2=7$$
$$x+3=\pm\sqrt{7}$$
$$x=-3\pm\sqrt{7}$$

5.

$x^2-14x=-48$ To complete the square: $\left(\frac{14}{2}\right)^2=49$.

$x^2-14x+49-49=-48$ underlined piece is a perfect square

$$(x-7)^2=1$$
$$x-7=\pm\sqrt{1}$$
$$x=7\pm1$$
$$x=7+1 \text{ or } 7-1$$
$$x=8 \text{ or } 6$$

6.

$$\frac{x^2}{3} - \frac{x}{3} = 3 \text{ We MUST have a coefficient of 1 in front of the } x^2 \text{ before we complete the square.}$$

$$x^2 - x = 9$$

$$x^2 - 1x = 9 \text{ To complete the square: } \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

$$x^2 - x + \frac{1}{4} - \frac{1}{4} = 9 \text{ underlined piece is a perfect square}$$

$$\left(x - \frac{1}{2}\right)^2 = 9 + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{37}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{37}{4}}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{37}}{2}$$

7.

$$2y^2 - y = 15$$

$$y^2 - \frac{1}{2}y = \frac{15}{2} \text{ To complete the square: } \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$$

$$y^2 - \frac{1}{2}y + \frac{1}{16} - \frac{1}{16} = \frac{15}{2}$$

$$\left(y - \frac{1}{4}\right)^2 - \frac{1}{16} = \frac{15}{2}$$

$$\left(y - \frac{1}{4}\right)^2 = \frac{1}{16} + \frac{120}{16}$$

$$y - \frac{1}{4} = \pm\sqrt{\frac{121}{16}}$$

$$y = \frac{1}{4} \pm \frac{11}{4}$$

$$y = \frac{1}{4} + \frac{11}{4} \text{ or } \frac{1}{4} - \frac{11}{4}$$

$$y = 3 \text{ or } -\frac{5}{2}$$

8. Solve by completing the square.

$$x^2 - 2x = -7 \text{ To complete the square: } \left(\frac{2}{2}\right)^2 = 1.$$
$$x^2 - 2x + 1 - 1 = -7$$
$$(x-1)^2 - 1 = -7$$
$$x - 1 = \pm\sqrt{-6}$$
$$x - 1 = \pm\sqrt{6}i$$
$$x = 1 \pm \sqrt{6}i$$

9. Solve by completing the square.

$$3x^2 + 8x + 3 = 2$$
$$x^2 + \frac{8}{3}x + 1 = \frac{2}{3}$$
$$x^2 + \frac{8}{3}x = -\frac{1}{3} \text{ To complete the square: } \left(\frac{1}{2}\cdot\frac{8}{3}\right)^2 = \frac{16}{9}.$$
$$x^2 + \frac{8}{3}x + \frac{16}{9} - \frac{16}{9} = -\frac{1}{3}$$
$$\left(x + \frac{4}{3}\right)^2 = \frac{16}{9} - \frac{1}{3}$$
$$\left(x + \frac{4}{3}\right)^2 = \frac{13}{9}$$
$$x + \frac{4}{3} = \pm\sqrt{\frac{13}{9}}$$
$$x = -\frac{4}{3} \pm \frac{\sqrt{13}}{3}$$

10. Sketch $f(x) = 2x^2 + 2x - 4$.

Since $a = 2 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{2}{2(2)} = -\frac{1}{2}$$

$$y = f(-1/2) = 2(-1/2)^2 + 2(-1/2) - 4 = -\frac{9}{2}$$

$$x\text{-intercepts: } 2x^2 + 2x - 4 = 0$$

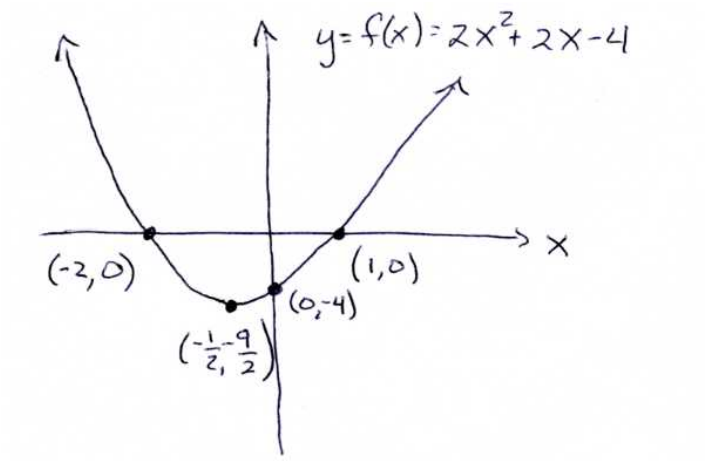
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-2 \pm 6}{4}$$

$$x = 1 \text{ or } x = -2$$

$$y\text{-intercept: } f(0) = 2(0)^2 + 2(0) - 4 = -4$$

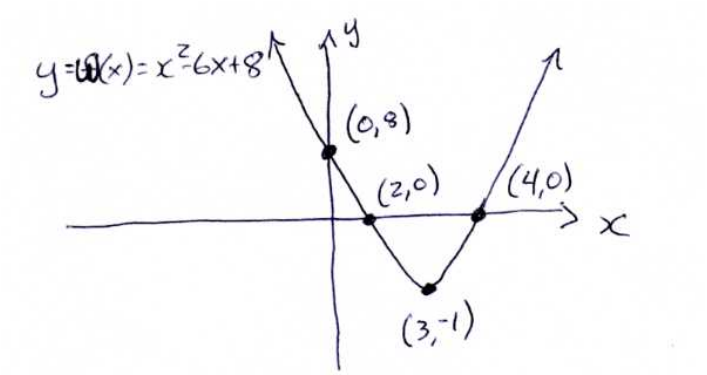


11. Sketch $w(x) = x^2 - 6x + 8$.

Since $a = 1 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3$$
$$y = f(3) = (3)^2 - 6(3) + 8 = -1$$

$$x\text{-intercepts: } x^2 - 6x + 8 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$
$$x = \frac{6 \pm 2}{2}$$
$$x = 4 \text{ or } x = 2$$
$$y\text{-intercept: } f(0) = (0)^2 - 6(0) + 8 = +8$$

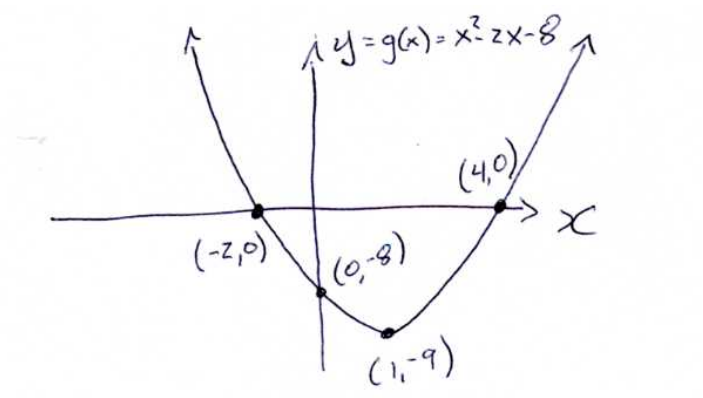


12. Sketch $g(x) = x^2 - 2x - 8$.

Since $a = 1 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = -1$$
$$y = f(-1) = (-1)^2 - 2(-1) - 8 = -9$$

$$x\text{-intercepts: } x^2 - 2x - 8 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$
$$x = \frac{2 \pm 6}{2}$$
$$x = 4 \text{ or } x = -2$$
$$y\text{-intercept: } f(0) = (0)^2 - 2(0) - 8 = -8$$



13. Sketch $r(x) = -3x^2 + 6x - 4$.

Since $a = -3 < 0$, quadratic opens down

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{6}{2(-3)} = 1$$
$$y = f(1) = -3(1)^2 + 6(1) - 4 = -1$$

$$x\text{-intercepts: } -3x^2 + 6x - 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(-3)(-4)}}{2(-3)}$$

since $b^2 - 4ac = -12 < 0$, there are no x-intercepts

$$y\text{-intercept: } f(0) = -3(0)^2 + 6(0) - 4 = -4$$