Questions

1. Solve $\cos 2x + \cos x = 0$ algebraically for exact solutions in the interval $[0, 2\pi)$.

2. Solve $\cos 2x + \sin x = 0$ algebraically for exact solutions in the interval $[0, 2\pi)$.

Solutions

1. Solve $\cos 2x + \cos x = 0$ algebraically for exact solutions in the interval $[0, 2\pi)$.

$$\begin{aligned}
\cos 2x + \cos x &= \cos^2 x - \sin^2 x + \cos x \\
&= \cos^2 x - (1 - \cos^2 x) + \cos x \\
&= 2\cos^2 x + \cos x - 1 = 0
\end{aligned}$$

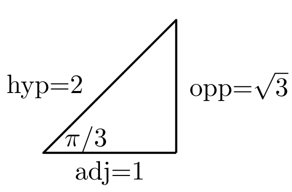
Let $y = \cos x$. Then

$$\begin{aligned}
\cos 2x + \cos x &= 2\cos^2 x + \cos x - 1 = 0 \\
&= 2y^2 + y - 1 = 0 \\
y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-1 \pm \sqrt{1+8}}{4} \\
&= \frac{-1 \pm 3}{4} \\
&= \frac{2}{4} \text{ or } \frac{-4}{4} = \frac{1}{2} \text{ or } -1
\end{aligned}$$

So we must solve $y = \cos x = 1/2$ and $y = \cos x = -1$.

The equation $\cos x = -1$ has a solution of π in the interval $[0, 2\pi)$.

The equation $\cos x = \text{adj/hyp} = 1/2$ corresponds to one of our special triangles:



So the solution is $\pi/3$. There is also a solution at $2\pi - \pi/3 = 5\pi/3$ in the interval $[0, 2\pi)$ (the solution in Quadrant IV).

The solutions to $\cos 2x + \cos x = 0$ in the interval $[0, 2\pi)$ are $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$.

2. Solve $\cos 2x + \sin x = 0$ algebraically for exact solutions in the interval $[0, 2\pi)$.

$$\begin{aligned} \cos 2x + \sin x &= \cos^2 x - \sin^2 x + \sin x \\ &= 1 - \sin^2 x - \sin^2 x + \sin x \\ &= -2\sin^2 x + \sin x + 1 = 0 \end{aligned}$$

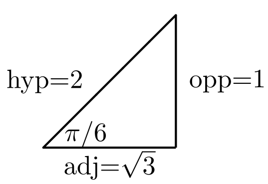
Let $y = \sin x$. Then

$$\begin{aligned} \cos 2x + \sin x &= -2\sin^2 x + \sin x + 1 = 0 \\ &= -2y^2 + y + 1 = 0 \\ y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1+8}}{-4} = \frac{-1 \pm 3}{-4} \\ &= \frac{2}{-4} \text{ or } \frac{-4}{-4} = -\frac{1}{2} \text{ or } 1 \end{aligned}$$

So we must solve $y = \sin x = -1/2$ and $y = \sin x = 1$.

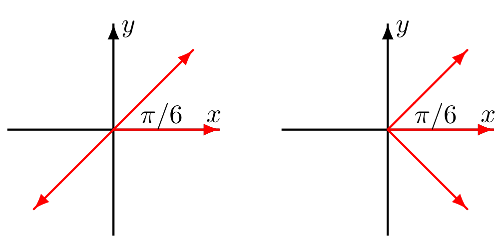
The equation $\sin x = 1$ has a solution of $\pi/2$ in the interval $[0, 2\pi)$.

The equation $\sin x = \text{opp}/\text{hyp} = 1/2$ corresponds to one of our special triangles:



So the solution to $\sin x = 1/2$ is $\pi/6$. We need to do a Quadrant analysis to get the solution to $\sin x = -1/2$

Since the sine is negative, that means we get solutions in Quadrants III and IV.



Quadrant III: $\pi/6 + \pi = 7\pi/6$. Quadrant IV: $2\pi - \pi/6 = 11\pi/6$.

The solutions to $\cos 2x + \sin x = 0$ in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$.