Questions

1. Find the exact value without a calculator for $\sin(\arctan 1)$.

2. Express $-\csc^2(\cot^{-1} x)$ as an algebraic expression involving no trigonometric functions.

3. Find the algebraic expression equivalent to the expression $\sin(\arccos 3x)$.

4. What is the value of $\arccos(-\sqrt{3}/2)$?

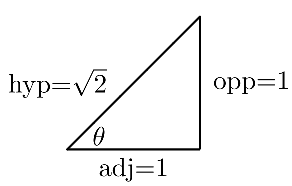
Solutions

1. Find the exact value without a calculator for $\sin(\arctan 1)$.

To simplify this we need to know the value of $\theta = \arctan 1$.

This means $\tan\theta = 1 = \dfrac{1}{1} = \dfrac{\text{opp}}{\text{adj}}$.

Construct a reference triangle



The length of the hypotenuse was found using the Pythagorean theorem

$$\text{hyp} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}.$$

$$\sin\big(\arctan 1\big) = \sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}.$$

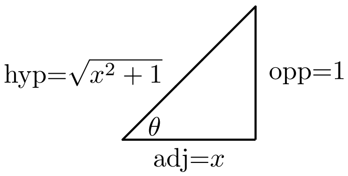
Notice the angle $\theta = \pi/4$ is one of our special angles, but we did not need to figure that out to solve this problem.

2. Express $-\csc^2(\cot^{-1} x)$ as an algebraic expression involving no trigonometric functions.

To simplify this we need to know the value of $\theta = \cot^{-1} x$.

This means $\cot\theta = x = \dfrac{x}{1} = \dfrac{\text{adj}}{\text{opp}}$.

Construct a reference triangle



The length of the hypotenuse was found using the Pythagorean theorem hyp $=\sqrt{1^2+x^2}=\sqrt{1+x^2}$.

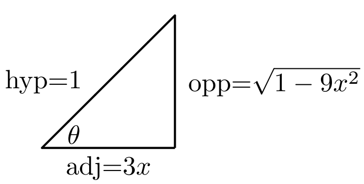
$$\begin{aligned}
\csc(\cot^{-1}x) &= \csc\theta \\
&= \frac{\text{hyp}}{\text{opp}} \\
&= \frac{\sqrt{x^2+1}}{1} \\
&= \sqrt{x^2+1} \\
-\csc^2(\cot^{-1}x) &= -(\sqrt{x^2+1})^2 \\
&= -(x^2+1) = -x^2-1
\end{aligned}$$

3. Find the algebraic expression equivalent to the expression $\sin(\arccos 3x)$.

To simplify this we need to know the value of $\theta = \arccos 3x$, then we can find $\sin\theta = \sin(\arccos 3x)$.

This means $\cos\theta = 3x = \dfrac{3x}{1} = \dfrac{\text{adj}}{\text{hyp}}$.

Construct a reference triangle



The length of the hypotenuse was found using the Pythagorean theorem

$$\text{hyp} = \sqrt{1^2-(3x)^2} = \sqrt{1-9x^2}.$$

$$\sin\big(\arccos 3x\big) = \sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{1-9x^2}}{1} = \sqrt{1-9x^2}.$$

4. What is the value of $\arccos(-\sqrt{3}/2)$?

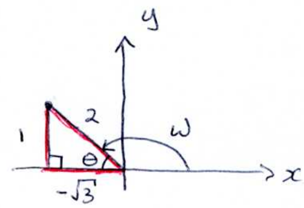
Let $\omega = \arccos\left(-\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \cos\omega = -\frac{\sqrt{3}}{2} = \frac{x}{r}$

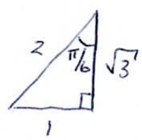
$\Rightarrow x = -\sqrt{3}$

$r = 2$

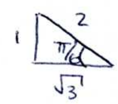
$y = \sqrt{r^2 - x^2} = 1$



Compare to



(redraw if needed in same orientation)



So $\theta = \pi/6$

$\omega = \pi - \pi/6 = 5\pi/6$

$\Rightarrow \boxed{\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}}$

I strongly feel being able to work things like this out using our knowledge of trigonometry and the special triangles is vastly superior to memorizing the values of trig functions on the unit circle.