

Questions

1. Describe the graph of the function in terms of basic trigonometric functions. Locate the vertical asymptotes and sketch two periods of the function.

$$y = 3 \tan(x/2)$$

2. Solve the equation $\csc x = 1$ in the interval $2\pi \leq x \leq 5\pi/2$. You should not need a calculator to solve this problem.

3. Solve the equation $\sec x = 2$ in the interval $-\pi/2 \leq x \leq \pi/2$. You should not need a calculator to solve this problem.

4. Solve the equation $\cot t = -\sqrt{3}$ for all possible values of t .

Solutions

1. Describe the graph of the function in terms of basic trigonometric functions. Locate the vertical asymptotes and sketch two periods of the function.

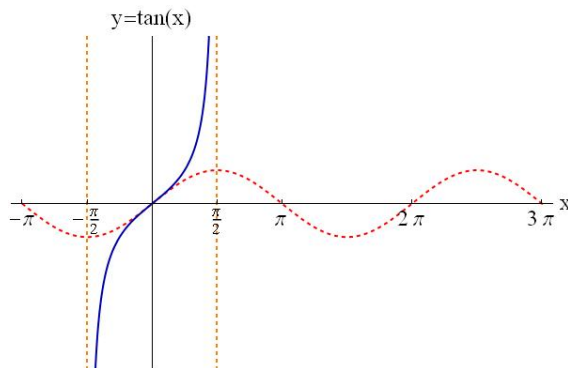
$$y = 3 \tan(x/2)$$

The basic trig function is $y = \tan x$. It has period π . Therefore, this new function will complete one period when

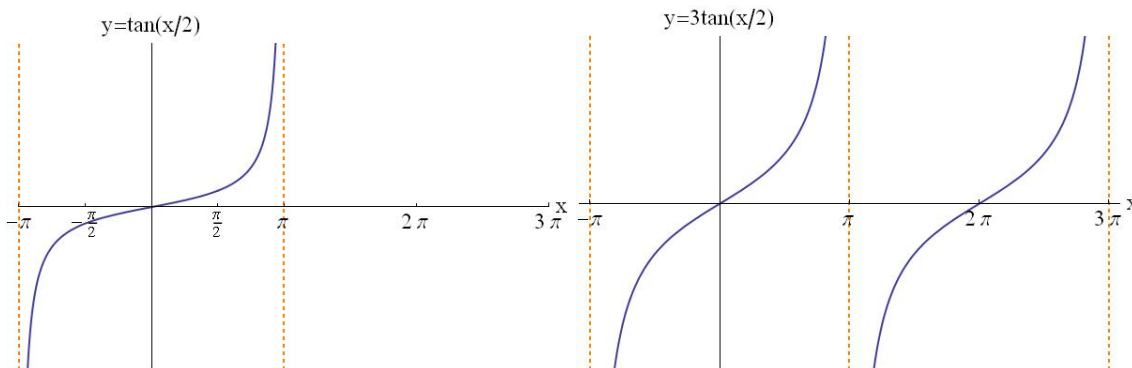
$$\begin{aligned} -\pi/2 &\leq x/2 \leq \pi/2 \\ -\pi &\leq x \leq \pi \end{aligned}$$

It will also have vertical stretch of 3 units compared to $y = \tan x$.

We construct the sketch of $\tan x$ by remembering how it relates to the sine function. The sine function is included in the sketch to help us.



Notice that we only drew one period of the sine and tangent function in the above sketch. Now we can do the vertical stretch and the horizontal stretch (left) and then two periods of final function (right):

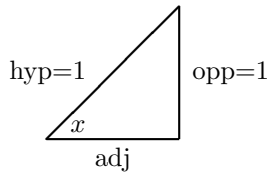


We see that the vertical asymptotes of $y = 3 \tan(x/2)$ occur at $x = k\pi$, where $k = 0, \pm 1, \pm 2, \pm 3, \dots$

2. Solve the equation $\csc x = 1$ in the interval $2\pi \leq x \leq 5\pi/2$. You should not need a calculator to solve this problem.

Let's construct a reference triangle to help us solve this problem. Since we are told not to use a calculator, we expect that the reference triangle will be one of the two special triangles, a 45-45-90 or a 30-60-90.

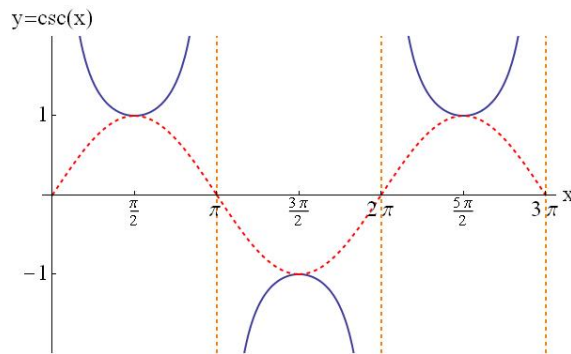
$$\csc x = 1 = \frac{1}{1} = \frac{1}{\sin x} = \frac{\text{hyp}}{\text{opp}}$$



The adjacent side has length zero! This isn't one of our special triangles, it is a quadrantal angle! So we can still solve this, but it isn't one of our special triangles after all.

The cosecant equal to one means the sine is equal to one. The angle with sine equal to one is $\pi/2$, so $x = \pi/2$.

However, we know that we can add (or subtract) 2π to this angle and get another solution to the equation. So $x = \pi/2 + 2\pi = 5\pi/2$ is also a solution. Since this is in the interval we want, this is our solution. Here is a sketch of the cosecant function so you can see the solution



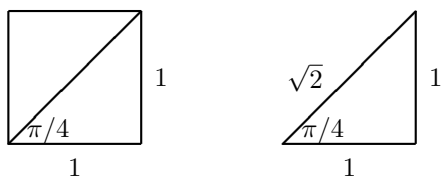
3. Solve the equation $\sec x = 2$ in the interval $-\pi/2 \leq x \leq \pi/2$. You should not need a calculator to solve this problem.

Use what we know about secant to get started by converting into something with a cosine.

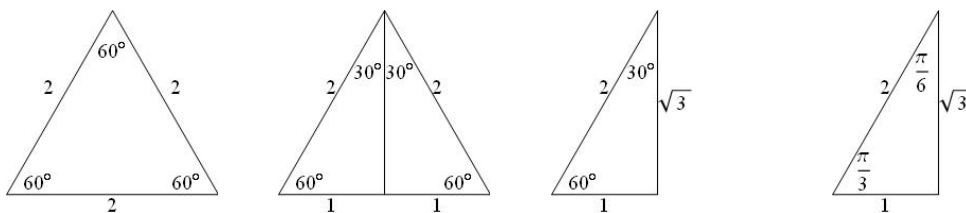
$$\sec x = \frac{1}{\cos x} = 2 \Rightarrow \cos x = \frac{1}{2}.$$

This is one of our special angles (if you are told you don't need a calculator, or to find something exactly, it is usually because it involves an angle in one of the special triangles).

A 45-45-90 Triangle



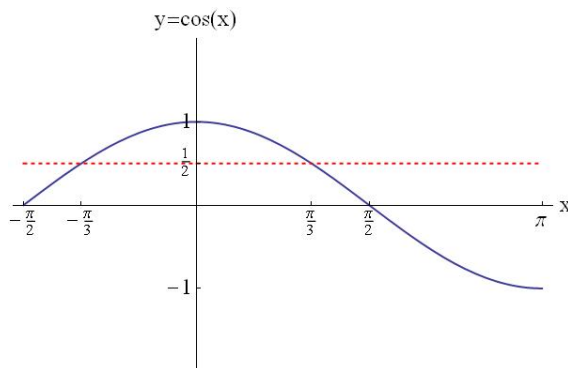
A 30-60-90 Triangle



The $\pi/3$ angle gives $\cos(\pi/3) = 1/2$. So $x = \pi/3$.

We were asked to find solutions in the interval $-\pi/2 \leq x \leq \pi/2$, so there is also a solution at $x = -\pi/3$.

Here is a sketch of the cosine function so you can see the solution



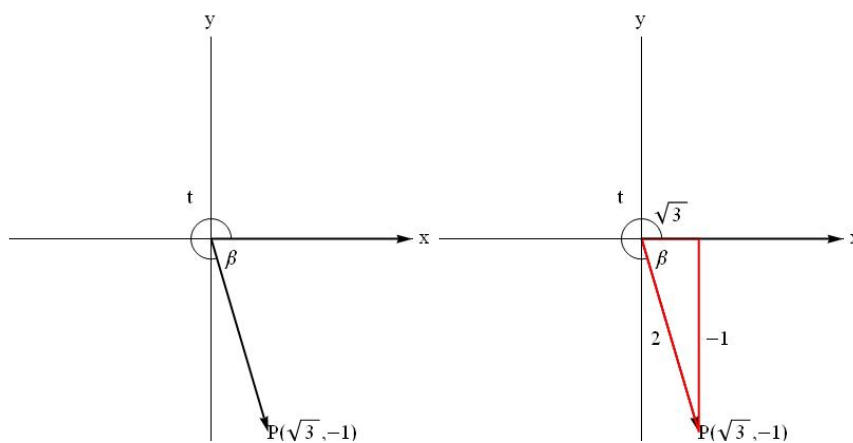
4. Solve the equation $\cot t = -\sqrt{3}$ for all possible values of t .

Let's use the coordinate axes method to solve the problem rather than reference triangles.

When the cotangent of t is less than zero we must be in either Quadrant II or Quadrant IV (where $x = r \cos t$ and $y = r \sin t$ have different signs). With no other information provided, this is all we can say.

Let's assume we are in the fourth quadrant, and see what happens. Quadrant IV means $y < 0$.

$$\cot t = -\sqrt{3} = \frac{\sqrt{3}}{-1} = \frac{x}{y}$$



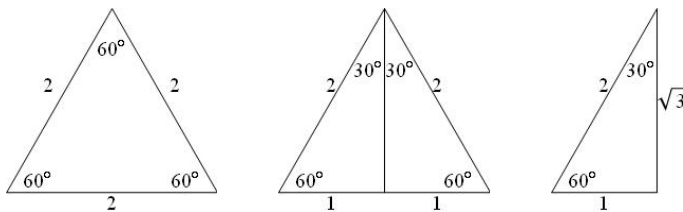
In the second diagram, I have labelled the lengths of the sides, being careful to indicate quantities which are less than zero based on the quadrant the point P is in.

Also, we have found r using the Pythagorean theorem.

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2.$$

We see from the diagram that this is one of our special triangles, a 30-60-90 triangle.

This triangle results from the equilateral triangle:



We see the angle $\beta = 30^\circ = \pi/6$ radians (although, this might be a bit hard to see since my coordinate axes diagram was not drawn to scale—if you have trouble seeing which angle it is, try drawing as much as you can to the correct scale).

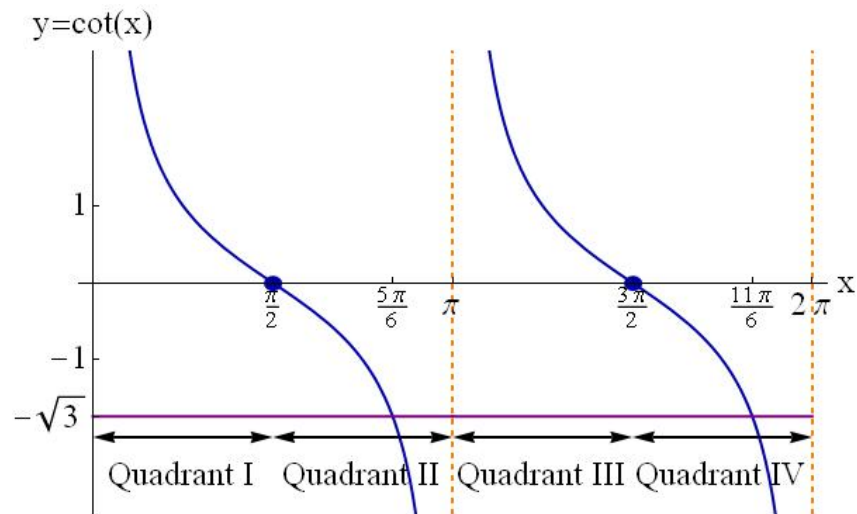
The angle we seek is $t = 2\pi - \beta = 2\pi - \pi/6 = 11\pi/6$.

Since the cotangent function has period π , all the solutions t to the equation $\cot t = -\sqrt{3}$ are

$$t = \frac{11\pi}{6} + k\pi, \quad k = 0, \pm 1, \pm 2, \pm 3, \dots$$

Notice that $k = \pm 1$ gives us the solution ($t = 5\pi/6$) we would have found had we chosen to work in Quadrant II.

Here is a sketch of the cotangent function so you can see the solution makes sense



Notice that in the solution, I used both a reference triangle as well as the x, y, r formalism. Use whatever seems best at the time, and ensure that your solution is mathematically consistent.