These problems give you a chance to practice your knowledge of the twelve basic functions, as well as graphical transformations. Sketch the functions by hand as part of your solution.

Questions

1. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = f(-x)$ are related.

2. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = -f(x)$ are related.

3. Explain how the graphs of $y = f(x) = x^3$ and $y = f(x) - 3$ are related.

4. Explain how the graphs of $y = f(x) = 1/x$ and $y = f(x - 2)$ are related.

5. Explain how the graphs of $y = f(x) = x$ and $y = 2f(x)$ are related.

6. Explain how the graphs of $y = f(x) = x$ and $y = f(3x)$ are related.

7. Sketch $y = \dfrac{12}{x - 7}$. State the domain and range.

8. Sketch $y = -\dfrac{3}{x + 2} - 1$. State the domain and range.

9. Sketch $y = -4|2x| + 1$. State the domain and range.

10. Sketch $y = -3\sqrt{x - 2} - 1$. State the domain and range.

Solutions

My solutions will include sketches using *Mathematica* so you can best see how the two functions are related. You should be able to draw these sketches by hand.

1. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = f(-x)$ are related.

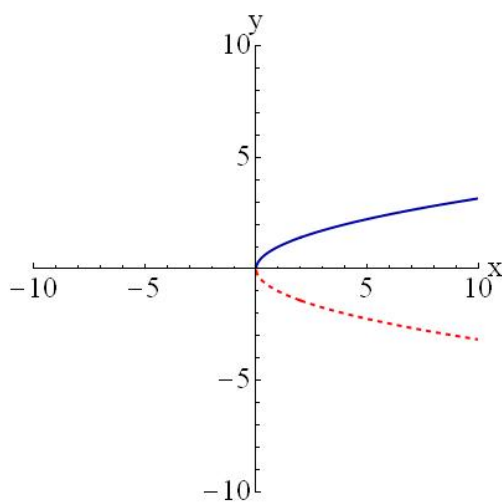The graph of $y = \sqrt{x}$ is a square root function (blue).

The graph of $y = f(-x)$ is modified inside the f (so it is a horizontal change) and since it is $f(-x)$ this is a reflection about the y-axis (red).
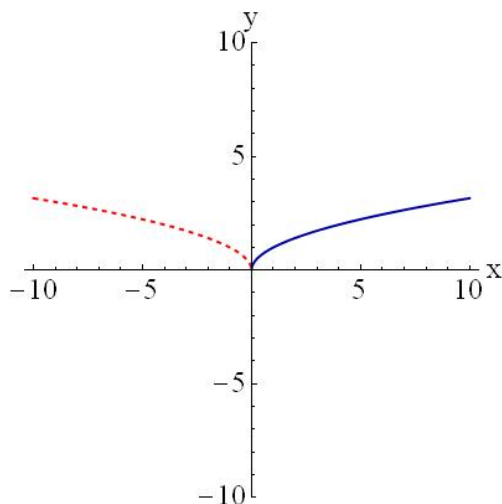
2. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = -f(x)$ are related.

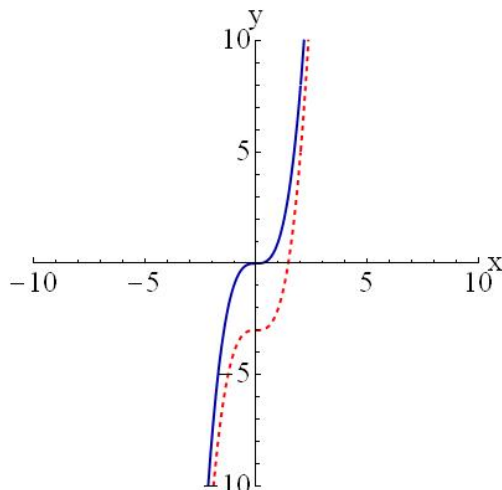The graph of $y = \sqrt{x}$ is a square root function (blue).

The graph of $y = -f(x)$ is modified outside the f (so it is a vertical change) and since it is $-f(x)$ this is a reflection about the x-axis (red).

3. Explain how the graphs of $y = f(x) = x^3$ and $y = f(x) - 3$ are related.
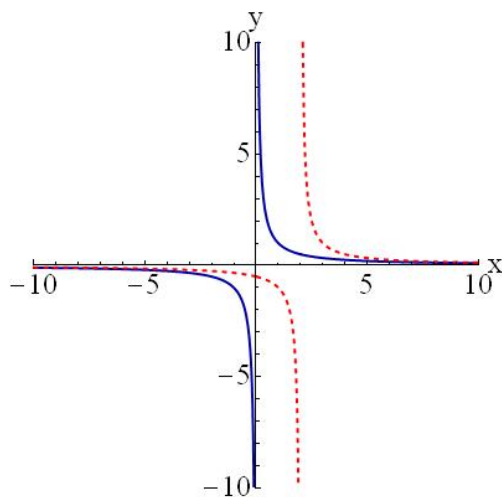
The graph of $y = x^3$ is a cube function (blue).

The graph of $y = f(x) - 3$ is modified outside the f (so it is a vertical change) and since it is $f(x) - 3$ this is shifted down three units (red).

4. Explain how the graphs of $y = f(x) = 1/x$ and $y = f(x - 2)$ are related.
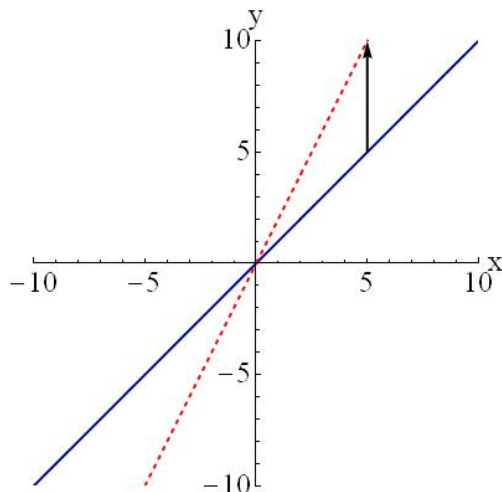
The graph of $y = 1/x$ is a reciprocal function (blue).

The graph of $y = f(x - 2)$ is modified inside the f (so it is a horizontal change) and since it is $f(x - 2)$ this is shifted to the right two units (red).

5. Explain how the graphs of $y = f(x) = x$ and $y = 2f(x)$ are related.

The graph of $y = x$ is a linear function (blue).

The graph of $y = 2f(x)$ is modified outside the f (so it is a vertical change) and since it is $2f(x)$ this is stretched vertically by two units (red). The arrow shows the vertical stretch of the point $(5, 5)$ to the point $(5, 10)$ on the new graph.

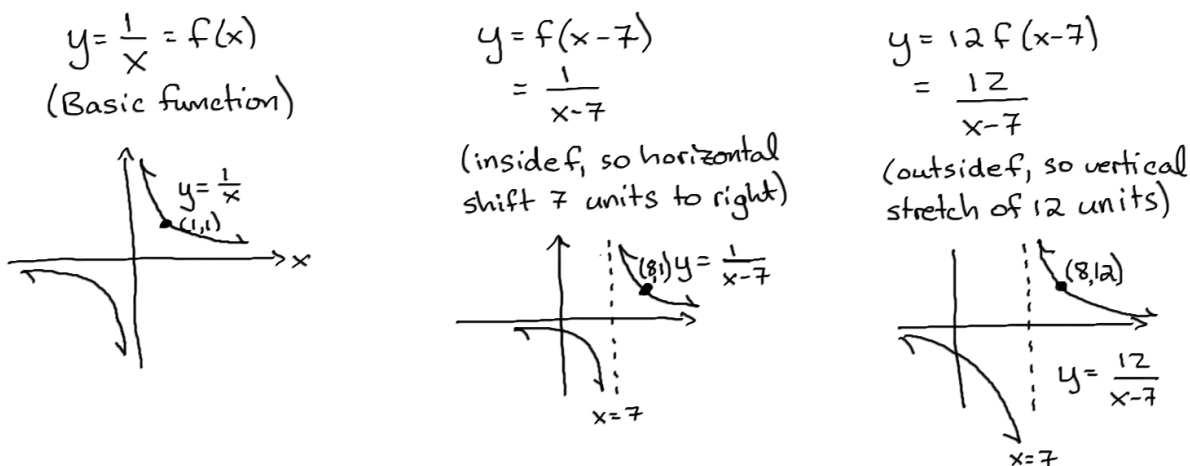**6.** Explain how the graphs of $y = f(x) = x$ and $y = f(3x)$ are related.

The graph of $y = x$ is a linear function (blue).

The graph of $y = f(3x)$ is modified inside the f (so it is a horizontal change) and since it is $f(3x)$ this is compressed horizontally by three units (red). The arrow shows the horizontal compression of the point $(6, 6)$ to the point $(2, 6)$ on the new graph.
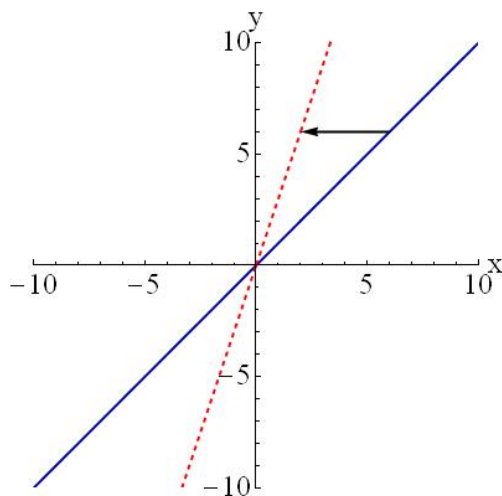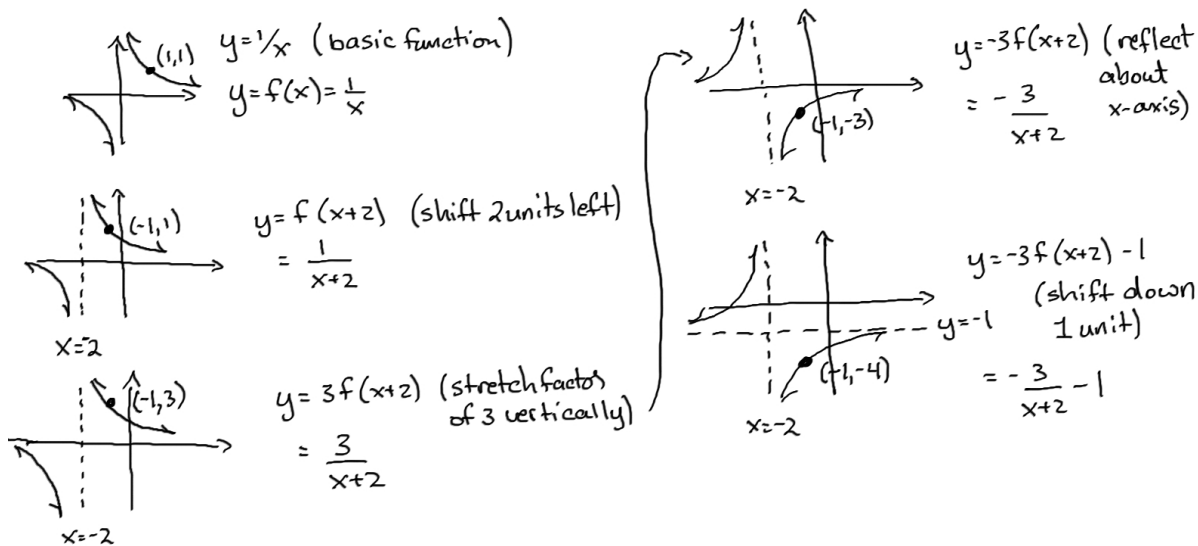
7. Sketch $y = \dfrac{12}{x-7}$. State the domain and range.

I am tracking what happens to one point as we make the transformations.

$$y = \frac{1}{x} = f(x)$$
(Basic function)

$$y = f(x-7)$$
$$= \frac{1}{x-7}$$
(inside f, so horizontal shift 7 units to right)

$$y = 12 f(x-7)$$
$$= \frac{12}{x-7}$$
(outside f, so vertical stretch of 12 units)

From the sketch, the domain is $x \in (-\infty, 7) \cup (7, \infty)$ and the range is $y \in (-\infty, 0) \cup (0, \infty)$.

8. Sketch $y = -\dfrac{3}{x+2} - 1$. State the domain and range.

$y = \frac{1}{x}$ (basic function)
$y = f(x) = \frac{1}{x}$

$y = f(x+2)$ (shift 2 units left)
$= \frac{1}{x+2}$

$y = 3f(x+2)$ (stretch factor of 3 vertically)
$= \frac{3}{x+2}$

$y = -3f(x+2)$ (reflect about x-axis)
$= -\frac{3}{x+2}$

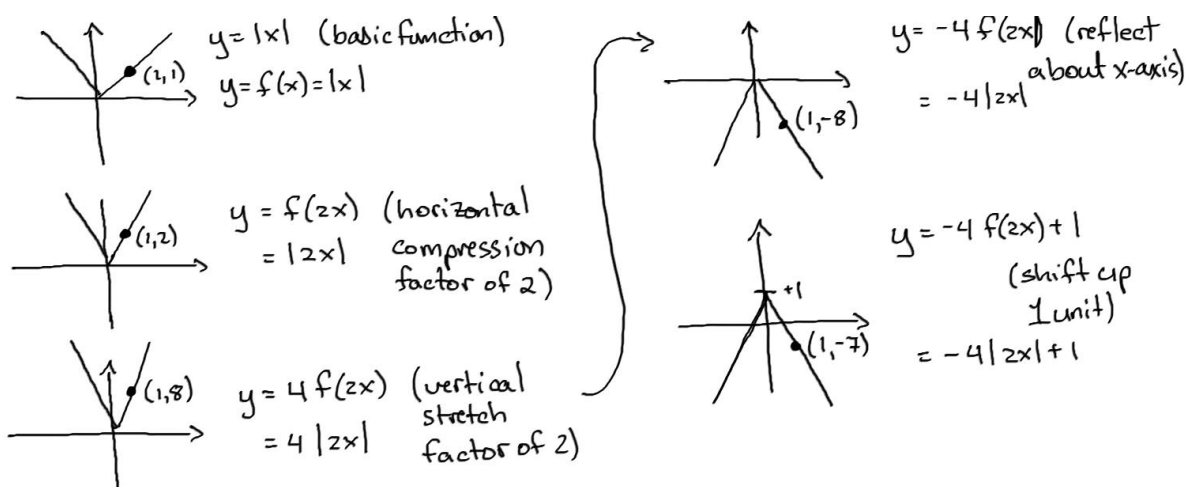$y = -3f(x+2) - 1$ (shift down 1 unit)
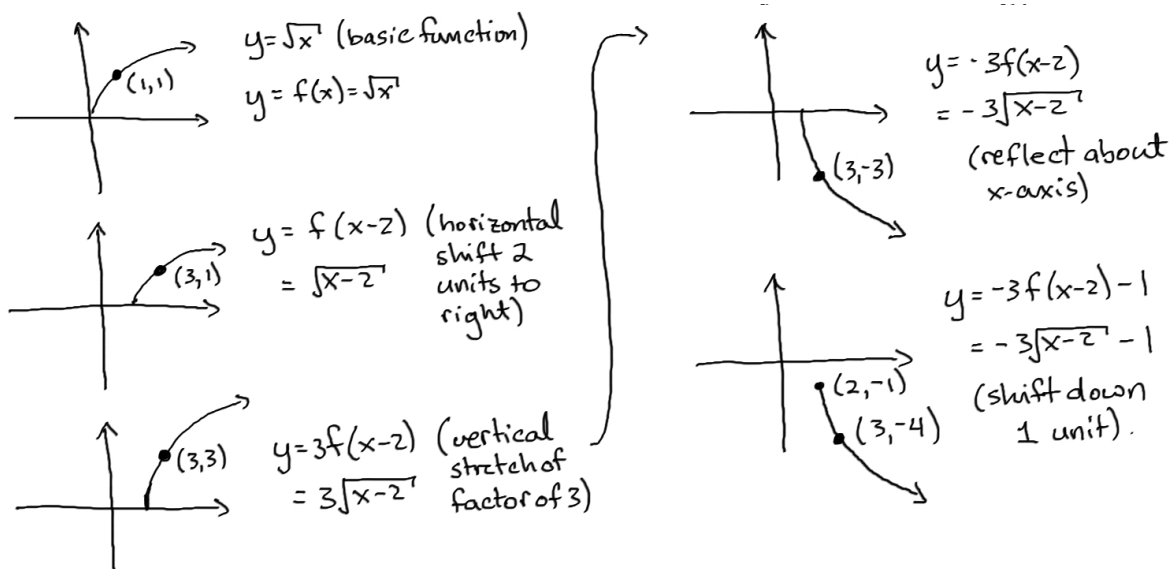$= -\frac{3}{x+2} - 1$

From the sketch, the domain is $x \in (-\infty, -2) \cup (-2, \infty)$ and the range is $y \in (-\infty, -1) \cup (-1, \infty)$.

9. Sketch $y = -4|2x| + 1$. State the domain and range.

$y = |x|$ (basic function)
$y = f(x) = |x|$
$(1,1)$

$y = f(2x)$ (horizontal compression factor of 2)
$= |2x|$
$(1,2)$

$y = 4f(2x)$ (vertical stretch factor of 2)
$= 4|2x|$
$(1,8)$

$y = -4f(2x)$ (reflect about x-axis)
$= -4|2x|$
$(1,-8)$

$y = -4f(2x) + 1$ (shift up 1 unit)
$= -4|2x| + 1$
$(1,-7)$

From the sketch, the domain is $x \in (-\infty, \infty)$ and the range is $y \in (-\infty, 1]$.

10. Sketch $y = -3\sqrt{x-2} - 1$. State the domain and range.

$y = \sqrt{x}$ (basic function)
$y = f(x) = \sqrt{x}$
$(1,1)$

$y = f(x-2)$ (horizontal shift 2 units to right)
$= \sqrt{x-2}$
$(3,1)$

$y = 3f(x-2)$ (vertical stretch of factor of 3)
$= 3\sqrt{x-2}$
$(3,3)$

$y = -3f(x-2)$
$= -3\sqrt{x-2}$
(reflect about x-axis)
$(3,-3)$

$y = -3f(x-2) - 1$
$= -3\sqrt{x-2} - 1$
(shift down 1 unit).
$(2,-1)$
$(3,-4)$

From the sketch, the domain is $x \in (2, \infty)$ and the range is $y \in (-\infty, -1]$.