Questions

1. Solve for x when $2\log_{10} x + \log_{10} 5 = 1$.

2. Solve for x if $\log_3(4x+6) - \log_3(x-1) = 2$.

3. Solve for x if $\log_{10} x + \log_{10}(2x+1) = 1$.

4. Solve the equation $\dfrac{500}{1+25e^{0.3x}} = 200$ algebraically.

5. Solve the equation $\log(x-2) + \log(x+5) = 2\log 3$ algebraically.

6. Solve the equation $\dfrac{e^x + e^{-x}}{2} = 4$ algebraically.

Hint: Solving this involves noticing that the equation can be rewritten to be quadratic in e^x.

7. Solve $\ln(x-10) + \ln(x+10) = 12$ for x.

Solutions

1. Solve for x when $2\log_{10} x + \log_{10} 5 = 1$.

You solve this by using the rules of logarithms and exponents. It can help you memorize the rules if you write the rule down every time you use it when solving a problem.

$$\begin{aligned}
2\log_{10} x + \log_{10} 5 &= 1 \text{ Use Power Rule: } p\log_b M = \log_b(M^p)\\
\log_{10}(x^2) + \log_{10} 5 &= 1 \text{ Use Product Rule: } \log_b M + \log_b N = \log_b(MN)\\
\log_{10}(5x^2) &= 1\\
10^{\log_{10}(5x^2)} &= 10^1 \text{ Use } b^{\ln_b(A)} = A\\
5x^2 &= 10 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}
\end{aligned}$$

We must check these solutions, since you can get extraneous solutions appearing.

Check $\sqrt{2}$:

$$\begin{aligned}
&2\log_{10}\sqrt{2} + \log_{10} 5 = 1\\
&\log_{10}(\sqrt{2}^2) + \log_{10} 5 = 1\\
&\log_{10} 2 + \log_{10} 5 = 1\\
&\log_{10}(2\times 5) = 1\\
&\log_{10}(10) = 1 \text{ True, so } x = \sqrt{x} \text{ is a solution.}
\end{aligned}$$

Check $-\sqrt{2}$:

$$2\log_{10}(-\sqrt{2}) + \log_{10} 5 = 1$$

We already have problems. The square root of a negative is not a real number (look at the graphs in Section 12.2, and you see that the logarithm is not defined for negative numbers). So $x = -\sqrt{2}$ is not a solution.

2. Solve for x if $\log_3(4x+6) - \log_3(x-1) = 2$.

$$\begin{aligned}
\log_3(4x+6) - \log_3(x-1) &= 2 \text{ Use Quotient Rule: } \log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)\\
\log_3\left(\frac{4x+6}{x-1}\right) &= 2 \text{ Take both sides as power of 3}\\
3^{\log_3\left(\frac{4x+6}{x-1}\right)} &= 3^2 \text{ Use } b^{\ln_b A} = A \text{ to simplify left hand side}\\
9 &= \frac{4x+6}{x-1} \text{ Now solve for } x\text{, using techniques from previous units}\\
9(x-1) &= 4x+6\\
9x-9 &= 4x+6\\
9x-4x &= 6+9\\
5x &= 15 \Rightarrow x = 3
\end{aligned}$$

Check:

$$\begin{aligned}
&\log_3(4(3)+6) - \log_3(3-1) = 2\\
&\log_3(18) - \log_3(2) = 2\\
&\log_3\left(\frac{18}{2}\right) = 2\\
&\log_3 9 = 2 \Rightarrow 3^2 = 9 \text{ which is True, so } x = 3 \text{ is a solution.}
\end{aligned}$$

3. Solve for x if $\log_{10} x + \log_{10}(2x+1) = 1$.

$$\begin{aligned}
\log_{10} x + \log_{10}(2x+1) &= 1 \text{ Use Product Rule: } \log_b M + \log_b N = \log_b(MN) \\
\log_{10}(x(2x+1)) &= 1 \\
10^{\log_{10}(x(2x+1))} &= 10^1 \text{ Use } 10^{\log A} = A \text{ to simplify left hand side} \\
10 &= x(2x+1) \text{ Now solve for } x\text{, using techniques from previous units} \\
10 &= 2x^2 + x \\
2x^2 + x - 10 &= 0 \text{ use quadratic formula} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 2, b = 1, c = -10 \\
x &= \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-10)}}{2(2)} \\
x &= \frac{-1 \pm \sqrt{81}}{4} \\
x &= \frac{-1 \pm 9}{4} \\
x &= \frac{-1-9}{4} \text{ or } \frac{-1+9}{4} \\
x &= \frac{-5}{2} \text{ or } 2
\end{aligned}$$

We exclude the negative solution, since it would lead to $\log_{10}(-5/2)$ which is not a real number.

Check $x = 2$:

$\log_{10} 2 + \log_{10}(2(2)+1) = 1$
$\log_{10} 2 + \log_{10} 5 = 1$
$\log_{10}(2 \times 5) = 1$
$\log_{10}(10) = 1 \Rightarrow 10^1 = 10$ True! So $x = 2$ is a solution.

4. Solve the equation $\dfrac{500}{1+25e^{0.3x}} = 200$ algebraically.

$$\begin{aligned}
\frac{500}{1+25e^{0.3x}} &= 200 \quad \text{(isolate the } e^{0.3x}\text{)} \\
\frac{1+25e^{0.3x}}{500} &= \frac{1}{200} \\
1+25e^{0.3x} &= \frac{500}{200} \\
25e^{0.3x} &= \frac{5}{2} - 1 \\
25e^{0.3x} &= \frac{3}{2} \\
e^{0.3x} &= \frac{3}{50} \quad \text{(take the natural logarithm of both sides)} \\
\ln e^{0.3x} &= \ln\left(\frac{3}{50}\right) \quad \text{(use the logarithm/exponential inverse function property)} \\
0.3x &= \ln\left(\frac{3}{50}\right) \\
x &= \frac{1}{0.3}\ln\left(\frac{3}{50}\right) \\
&= \frac{10}{3}\ln\left(\frac{3}{50}\right) \\
&\sim -9.37804
\end{aligned}$$

5. Solve the equation $\log(x-2) + \log(x+5) = 2\log 3$ algebraically.

$$\begin{aligned}
\log(x-2) + \log(x+5) &= 2\log 3 \\
\log((x-2)(x+5)) &= \log 3^2 \\
10^{\log(x-2)(x+5)} &= 10^{\log 9} \\
(x-2)(x+5) &= 9 \\
x^2 + 3x - 10 &= 9 \\
x^2 + 3x - 19 &= 0
\end{aligned}$$

Use the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-19)}}{2(1)} \\
&= \frac{-3 \pm \sqrt{9+76}}{2} \\
&= \frac{-3 \pm \sqrt{85}}{2}
\end{aligned}$$

So the solution is $x = (-3-\sqrt{85})/2 \sim -6.10977$ and $x = (-3+\sqrt{85})/2 \sim 3.10977$. However, one of these is extraneous. From the original equation, we must have $x - 2 \geq 0$ and $x + 5 \geq 0$ for the logarithms to be defined. Remember, $\log_b x$ is defined only for $x \geq 0$. These conditions are satisfied if $x \geq 2$, and the only solution to the problem is $x = (-3+\sqrt{85})/2 \sim 3.10977$.

6. Solve the equation $\dfrac{e^x + e^{-x}}{2} = 4$ algebraically.

Hint: Solving this involves noticing that the equation can be rewritten to be quadratic in e^x.

$$\begin{aligned}
\frac{e^x + e^{-x}}{2} &= 4 \\
e^x + e^{-x} &= 8 \\
e^x + e^{-x} - 8 &= 0 \\
e^x(e^x + e^{-x} - 8 &= 0) \\
e^{2x} + 1 - 8e^x &= 0 \\
e^{2x} - 8e^x + 1 &= 0 \\
e^{2x} - 8e^x + 1 &= 0
\end{aligned}$$

This is now a quadratic in e^x, let $z = e^x$, and we write

$$\begin{aligned}
z^2 - 8z + 1 &= 0 \\
z &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{+8 \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)} \\
&= \frac{8 \pm \sqrt{60}}{2} \\
&= 4 \pm \sqrt{15}
\end{aligned}$$

Therefore,

$$\begin{aligned}
z = e^x &= 4 \pm \sqrt{15} \\
\ln e^x &= \ln(4 \pm \sqrt{15}) \\
x &= \ln(4 \pm \sqrt{15})
\end{aligned}$$

Note that $\ln(e^x + e^{-x}) = \ln 8$ is a mathematical dead end. We can't solve for x from this.

7. Solve $\ln(x - 10) + \ln(x + 10) = 12$ for x.

$$\begin{aligned}
&\ln(x - 10) + \ln(x + 10) = 12 \text{ use } \ln A + \ln B = \ln(AB) \\
&\ln\left[(x - 10)(x + 10)\right] = 12 \\
&e^{\ln[(x-10)(x+10)]} = e^{12} \\
&e^{\ln[(x-10)(x+10)]} = e^{12} \text{ use fact that } e^{\ln A} = A \\
&(x - 10)(x + 10) = e^{12} \\
&x^2 - 100 = e^{12} \\
&x^2 = 100 + e^{12} \\
&x = \pm\sqrt{100 + e^{12}}
\end{aligned}$$

In original equation, for the logarithms to be defined we must have

$$x - 10 > 0 \text{ and } x + 10 > 0 \Rightarrow x > 10 \text{ and } x > -10 \Rightarrow x > 10$$

Since $x = -\sqrt{100 + e^{12}} < 0$, it is extraneous, and the only solution is $x = \sqrt{100 + e^{12}}$ which is greater than 10.