Questions

1. Given that $f(x) = 3x + 17$ and $g(x) = \dfrac{1}{1+x}$, compute $(f \circ g)(x)$ and $(g \circ f)(x)$. Do not worry about the domains.

2. If $f(x) = \sqrt{x-1}$ and $g(x) = x^4$, find the functions fg, $f - g$, $f + g$, f/g and each of their domains.

3. Find $(f \circ g)(3)$ and $(g \circ f)(-2)$ when $f(x) = x^2 - 1$ and $g(x) = 2x - 3$.

4. Find $f(g(x))$ and $g(f(x))$ if $f(x) = 1/(x-1)$ and $g(x) = \sqrt{x}$. State the domain of each.

5. Decompose the function $f(x) = \dfrac{1}{\cos x^3}$ so that each decomposed function is one of the twelve basic functions.

Solutions

1. Given that $f(x) = 3x + 17$ and $g(x) = \dfrac{1}{1+x}$, compute $(f \circ g)(x)$ and $(g \circ f)(x)$. Do not worry about the domains.

$$\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f(\frac{1}{1+x}) \\
&= 3(\frac{1}{1+x}) + 17 \\
&= \frac{3}{1+x} + 17 \\
(g \circ f)(x) &= g(f(x)) \\
&= g(3x+17) \\
&= \frac{1}{1+(3x+17)} \\
&= \frac{1}{3x+18)}
\end{aligned}$$

2. If $f(x) = \sqrt{x-1}$ and $g(x) = x^4$, find the functions $(fg)(x)$, $(f-g)(x)$, $(f+g)(x)$, $(f/g)(x)$ and each of their domains.
Domain of f is $x \in [1, \infty) = A$. Domain of g is $x \in (-\infty, \infty) = B$. Intersection $A \cap B = [1, \infty)$.

$$\begin{aligned}
(fg)(x) &= f(x)g(x) \\
&= \sqrt{x-1} \cdot x^4 \\
&= x^4\sqrt{x-1}, \text{ domain } [1, \infty) \\
(f-g)(x) &= f(x) - g(x) \\
&= \sqrt{x-1} - x^4, \text{ domain } [1, \infty) \\
(f+g)(x) &= f(x) + g(x) \\
&= \sqrt{x-1} + x^4, \text{ domain } [1, \infty) \\
(f/g)(x) &= f(x)/g(x) \\
&= \frac{\sqrt{x-1}}{x^4}, \text{ domain } [1, 0) \cup (0, \infty)
\end{aligned}$$

3. Find $(f \circ g)(3)$ and $(g \circ f)(-2)$ when $f(x) = x^2 - 1$ and $g(x) = 2x - 3$.

$$\begin{aligned} (f \circ g)(3) &= f(g(3)) \\ &= f(2(3) - 3) \\ &= f(3) \\ &= (3)^2 - 1 \\ &= 8 \end{aligned}$$

$$\begin{aligned} (g \circ f)(-2) &= g(f(-2)) \\ &= g((-2)^2 - 1) \\ &= g(3) \\ &= 2(3) - 3 \\ &= 3 \end{aligned}$$

4. Find $f(g(x))$ and $g(f(x))$ if $f(x) = 1/(x-1)$ and $g(x) = \sqrt{x}$. State the domain of each.

$$\begin{aligned} f(g(x)) &= f(\sqrt{x}) \\ &= \frac{1}{\sqrt{x} - 1} \end{aligned}$$

The domain of $f(g(x))$ is $x \in [0, 1) \cup (1, \infty)$.

$$\begin{aligned} g(f(x)) &= g\left(\frac{1}{x-1}\right) \\ &= \sqrt{\frac{1}{x-1}} \\ &= \frac{1}{\sqrt{x-1}} \end{aligned}$$

The domain of $g(f(x))$ is $x \in (1, \infty)$.

5. Decompose the function $f(x) = \dfrac{1}{\cos x^3}$ so that each decomposed function is one of the twelve basic functions.

Two evaluate this, you would cube, then take the cosine, then take the reciprocal. This gives you the decomposition. Pick $z(x) = x^3$, $h(x) = \cos x$, $g(x) = \frac{1}{x}$. Check:

$$f(x) = g(h(z(x))) = g(h(x^3)) = g(\cos x^3) = \frac{1}{\cos x^3}.$$