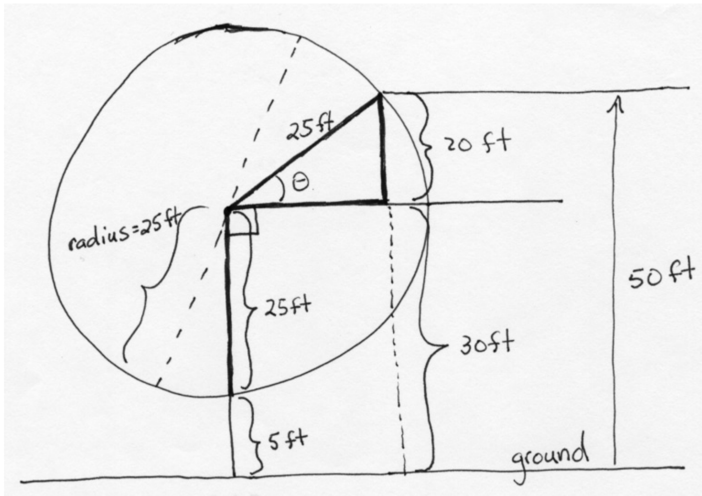
Concepts: Applications

This section just shows some applications of trigonometry, so the lecture notes serve as the Practice Problems.

Example A Ferris Wheel 50 ft in diameter makes one revolution every 40 seconds. If the center of the wheel is 30 ft above the ground, how long after reaching the low point is a rider 50 ft above the ground?

Solving this problem rests on constructing a good diagram. The radius of the wheel is 25 ft.

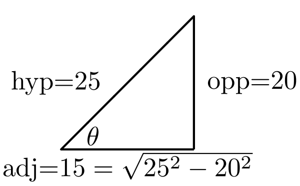


The wheel completes one revolution every 40 seconds.

The time it takes to complete $\theta + \pi/2$ revolutions (this will be a fraction of a complete revolution) will be the time it takes for a person to move from the bottom of the wheel to 50 ft above the ground. Letting this time be x, we have

$$\frac{2\pi}{\theta + \pi/2} = \frac{40}{x} \longrightarrow x = \frac{20}{\pi}(\theta + \pi/2).$$

We now need to determine the angle θ. There is a reference triangle in our diagram:

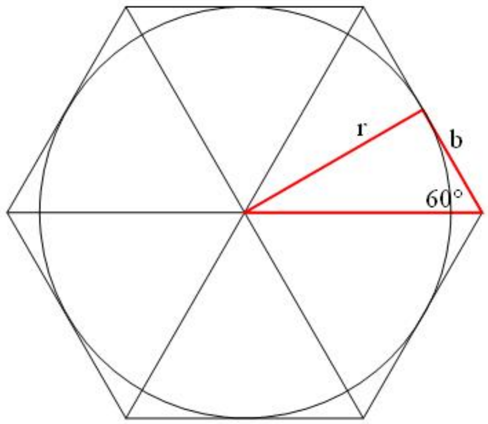


From the reference triangle, we see that $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{20}{25} = \dfrac{4}{5}$. Therefore, $\theta = \arcsin(4/5)$.

The time it takes for a person to go from the bottom of wheel to 50 ft above ground is $x = \dfrac{20}{\pi}(\arcsin(4/5) + \dfrac{\pi}{2}) \sim 15.9$ s.

This arcsine has to be evaluated using a calculator.

Example Determine the area of a hexagon with a circle of radius r inscribed within it in terms of r.



The interior angles of hexagon are 120°.

We see that the area of the hexagon is 12 times the area of the red triangle.

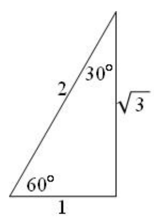
The area of the red triangle is $\frac{1}{2}$(base)(height).

We know the height is r.

I've labeled the base of the red triangle b, since we need to figure out what that is in terms of r.

From our trig:

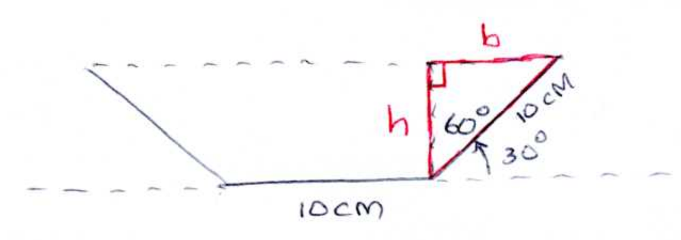
$$\tan 60^\circ = \frac{\text{opp}}{\text{adj}} = \frac{r}{b} \qquad \Rightarrow \qquad b = \frac{r}{\tan 60^\circ} = \frac{r}{\sqrt{3}/1} = \frac{r}{\sqrt{3}}.$$



So area of hexagon $= 12 \cdot \dfrac{1}{2} rb = 6 \cdot \dfrac{r}{\sqrt{3}} \cdot r = \dfrac{6r^2}{\sqrt{3}} = 2\sqrt{3}r^2$.

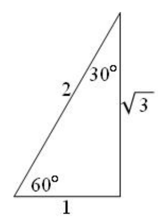
Note: You could also have determined $b = \dfrac{r}{\sqrt{3}}$ using similar triangles.

Example A rain gutter is made from a sheet of metal 30cm wide by bending up one-third of the sheet on each side by 30°. If the gutter is 40m long, how much water will it hold?



I've chosen to label the height of the red triangle in my sketch h and base b, since I will need both to determine the volume. Let's get h first, which makes me what to work with the cosine of 60° to determine h.

$$\cos 60° = \frac{\text{adj}}{\text{hyp}} = \frac{h}{10} \qquad \Rightarrow \qquad h = 10\cos 60° = 10 \cdot \frac{1}{2} = 5 \text{ cm}$$



Let's get the base using Pythagorean Theorem:

$$b = \sqrt{10^2 - 5^2} = \sqrt{75} = 5\sqrt{3} \text{ cm}.$$

All set–now the volume of the gutter will be the area of the cross section in the diagram, times the length of the gutter (40m = 400cm).

$$\text{Volume } = 400(2 \cdot \frac{1}{2}bh + 10h) = 400((5\sqrt{3})(10) + 10(10)) = 4000(5\sqrt{3} + 10) \sim 74{,}641\text{cm}^3$$