4452 Mathematical Modeling Lecture 5: *Mathematica* Worksheet

Here are some problems for you to work on. They are typical of the type of things that you will use *Mathematica* to do in the modeling course.

PROBLEM 1. Generate a sketch of the function $f(x, y) = (2x^3 + y^3 - 3xy)e^{-x^2-y^2}$. You can use the command **Plot3D** with the options **PlotPoints** and **PlotRange**. Adjust the options until you get a plot you are happy with.

Make sure you have assigned the above plot to a variable name. Then, use the **Show** command with the **ViewPoint** option to examine the object from many angles. You can add axeslabels with the option **AxesLabel**. At how many points does the function attain its *absolute* maximum and absolute minimum?

To find the extrema, we need to calculate the partial derivatives and solve the equations $f_x(x, y) = 0, f_y(x, y) = 0$ for the points (x, y). In *Mathematica* this can be done using:

```
eq1 = D[f[x, y], x] == 0
eq2 = D[f[x, y], y] == 0
Solve[{eq1, eq2}, {x, y}]
```

What kind of output did you get? How could you get an estimate of the solution that would be useful to you? Also, you can tell *Mathematica* to only provide solutions which are real-valued by loading the package RealOnly, which is done via `<<Miscellaneous‘RealOnly‘` (those quotes should be single backquotes!).

How many real-valued (x, y) points did you get? Do they all make sense if you look at your 3D sketch?

Generate a plot of the level curves of the function using **ContourPlot**. You may want to use the options **Contours** and **PlotPoints**. From this, can you deduce which points are local extrema, and which points are saddle points? There are four extrema and two saddlepoints. Was one of the saddle points a surprise? Can you determine the absolute max and the absolute min?

PROBLEM 2. Animate the graph of the 3D plot you created above. You can do this using the **Table** command, and changing the viewpoint. Once you have created a set of graphs to animate, you start the animation by double-clicking on one of the graphs. You can adjust the speed of the animation using the numeric keypad. A useful animation would be to circle around the plot and view it from different angles.

PROBLEM 3. Using the command **DSolve**, get *Mathematica* to solve the differential equation,

$$y'(x) = x^2 y(x) + y^2(x),$$

and the initial value problem

$$y'(x) = x^2 y(x) + y^2(x), \qquad y(0) = k.$$

What do the initial value problem solutions look like for $k = -1, 0, 1$?