14.6.55

$$f = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$df = \frac{\partial f}{\partial a} da + \frac{\partial f}{\partial b} db + \frac{\partial f}{\partial c} dc + \frac{\partial f}{\partial d} dd$$

$$\frac{\partial f}{\partial a} = d \qquad df = (d)\,da + (-c)\,db + (-b)\,dc + (a)\,dd$$

$$\frac{\partial f}{\partial b} = -c$$

$$\frac{\partial f}{\partial c} = -b$$

$$\frac{\partial f}{\partial d} = a$$

since this is the largest component ($|a| >> |b|$, etc) the determinant is most sensitive to changes in $\underline{d}$. A small change in d results in largest change to f, the determinant.

14.6.57

$$Q = \sqrt{\frac{2KM}{h}} \qquad P_0 = (2, 20, 0.05)$$

Sensitivity: compute total derivative

$$dQ\Big|_{P_0} = \frac{\partial Q}{\partial K}\Big|_{P_0} dK + \frac{\partial Q}{\partial M}\Big|_{P_0} dM + \frac{\partial Q}{\partial h}\Big|_{P_0} dh$$

$$\frac{\partial Q}{\partial K} = \sqrt{\frac{2M}{h}}\ \frac{1}{2} K^{-1/2} = \sqrt{\frac{M}{2hK}} \qquad \frac{\partial Q}{\partial K}\Big|_{P_0} = 10$$

$$\frac{\partial Q}{\partial M} = \sqrt{\frac{2K}{h}}\ \frac{1}{2} M^{-1/2} = \sqrt{\frac{K}{2hM}} \qquad \frac{\partial Q}{\partial M}\Big|_{P_0} = 1$$

$$\frac{\partial Q}{\partial h} = \sqrt{2KM}\ \left(-\frac{1}{2}\right) h^{-3/2} = -\sqrt{\frac{KM}{2h^3}} \qquad \frac{\partial Q}{\partial h}\Big|_{P_0} =$$

So a 1 unit change in K results in 10 unit change in Q.

1	M	1	Q
1	h	-400	Q

$\Rightarrow$ near P_0, Q is most sensitive to changes in h