13.3.8

$\vec{r}(t) = (t\sin t + \cos t)\hat{i} + (t\cos t - \sin t)\hat{j}$ , $\sqrt{2} \le t \le 2$.

$\vec{T} = \dfrac{\vec{v}}{|\vec{v}|}$ is unit tangent vector.

$\vec{v} = \dfrac{d}{dt}\vec{r}(t) = (\sin t + t\cos t - \sin t)\hat{i} + (\cos t - t\sin t - \cos t)\hat{j}$

$|\vec{v}| = \sqrt{(\cancel{\sin t} + t\cos t - \cancel{\sin t})^2 + (\cancel{\cos t} - t\sin t - \cancel{\cos t})^2}$

~~$= \sin^2 t + t\sin t\cos t - s$~~

$= \sqrt{t^2} = t$

$\vec{T} = \dfrac{t\cos t\,\hat{i} - t\sin t\,\hat{j}}{t} = \cos t\,\hat{i} - \sin t\,\hat{j}$

arc length $= \displaystyle\int_{\sqrt{2}}^{2} |v(t)|\,dt$

$= \displaystyle\int_{\sqrt{2}}^{2} t\,dt = \frac{t^2}{2}\Big|_{\sqrt{2}}^{2} = 2 - 1 = 1.$

13.3.10 $\vec{r}(t) = 12\sin t\,\hat{\imath} - 12\cos t\,\hat{\jmath} + 5t\,\hat{k}$

at point $(0,-12,0)$, $t = 0$.

Solve $\int_0^{-t} |\vec{v}|\,dt = 13\pi$ for $t$.

$\vec{v}(t) = 12\cos t\,\hat{\imath} + 12\sin t\,\hat{\jmath} + 5\,\hat{k}$

$|\vec{v}| = \sqrt{144\cos^2 t + 144\sin^2 t + 25} = \sqrt{169} = 13$

$\int_0^{-t} 13\,dt = 13\pi$

$13t\Big|_0^{-t} = 13\pi \Rightarrow -t = \pi \quad t = -\pi.$

At $t = -\pi$, we have $\vec{r}(-\pi) = 0\,\hat{\imath} + 12\,\hat{\jmath} - 5\pi\,\hat{k}$
so point is $(0,12,-5\pi)$.

13.3.12 $\vec{r}(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} \qquad \pi/2 \le t < \pi$

$$\vec{v}(t) = (-\sin t + \sin t + t\cos t)\hat{i} + (+\cos t - \cos t + t\sin t)\hat{j}$$
$$= t\cos t\,\hat{i} + t\sin t\,\hat{j}$$
$$|\vec{v}(t)| = \sqrt{t^2} = t$$
$$s = \int_0^t |\vec{v}(t)|\,dt$$
$$= \int_0^t t\,dt$$
$$s = \frac{t^2}{2}$$

From $\pi/2 \le t < \pi$, arc length $= s\Big|_{\pi/2}^{\pi} = \frac{t^2}{2}\Big|_{\pi/2}^{\pi} = \frac{\pi^2}{2} - \frac{\pi^2}{8} = \frac{3\pi^2}{8}$

13.3.18 a) $\vec{r}(t) = \cos 4t\,\hat{i} + \sin 4t\,\hat{j} + 4t\,\hat{k} \qquad 0 \le t \le \pi/2$

$$\vec{v}(t) = -4\sin 4t\,\hat{i} + 4\cos 4t\,\hat{j} + 4\hat{k}$$
$$|\vec{v}(t)| = \sqrt{16\sin^2 4t + 16\cos^2 4t + 16} = \sqrt{32} = 4\sqrt{2}$$
$$s = \int_0^{\pi/2} |\vec{v}(t)|\,dt = 4\sqrt{2}\int_0^{\pi/2} dt = 4\sqrt{2}\,\frac{\pi}{2} = 2\sqrt{2}\,\pi.$$

b) $\vec{r}(t) = \cos(t/2)\,\hat{i} + \sin(t/2)\,\hat{j} + t/2\,\hat{k} \qquad 0 \le t \le 4\pi$

$$\vec{v}(t) = -\tfrac{1}{2}\sin(t/2)\,\hat{i} + \tfrac{1}{2}\cos(t/2)\,\hat{j} + \tfrac{1}{2}\hat{k}$$
$$|\vec{v}(t)| = \sqrt{\tfrac{1}{4}\sin^2(t/2) + \tfrac{1}{4}\cos^2(t/2) + \tfrac{1}{4}} = \sqrt{1/2} = \frac{1}{\sqrt{2}}$$
$$s = \int_0^{4\pi} |\vec{v}(t)|\,dt = \frac{1}{\sqrt{2}}\int_0^{4\pi} dt = \frac{4\pi}{\sqrt{2}} = 2\sqrt{2}\,\pi$$

c) $\vec{r}(t) = \cos t\,\hat{i} - \sin t\,\hat{j} - t\,\hat{k}, \quad -2\pi \le t \le 0$

$$\vec{v}(t) = -\sin t\,\hat{i} - \cos t\,\hat{j} - \hat{k},$$
$$|\vec{v}(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$
$$s = \int_{-2\pi}^{0} |\vec{v}(t)|\,dt = \sqrt{2}\int_{-2\pi}^{0} dt = 2\sqrt{2}\,\pi.$$