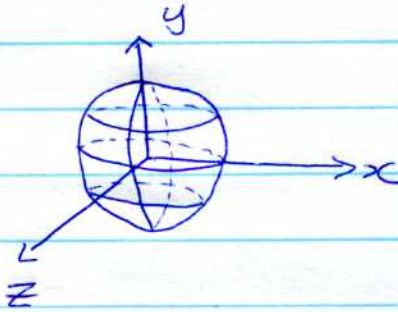
12.6.1 $x^2+y^2+4z^2=10$

This has the form of an ellipsoid.
Flatter in the z-direction.

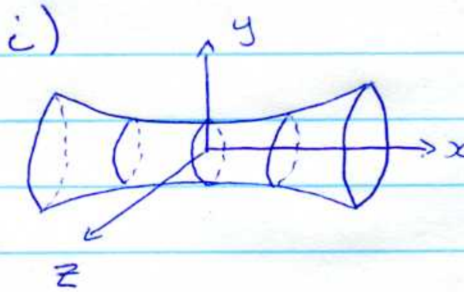
d)



12.6.2 $z^2+4y^2-4x^2=4$

This is a hyperboloid of 1 sheet.
opens along x-axis.

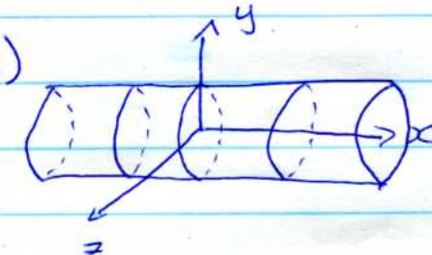
i)



12.6.3 $9y^2+z^2=16$

This is a cylinder.
opens along the x-axis.

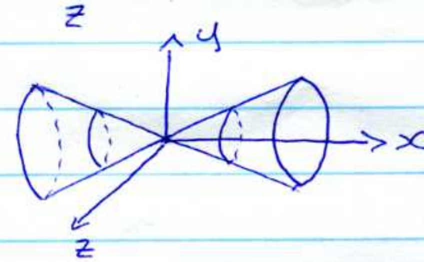
a)



12.6.4 $y^2+z^2=x^2$

This is an elliptical cone.
opens along the x-axis.

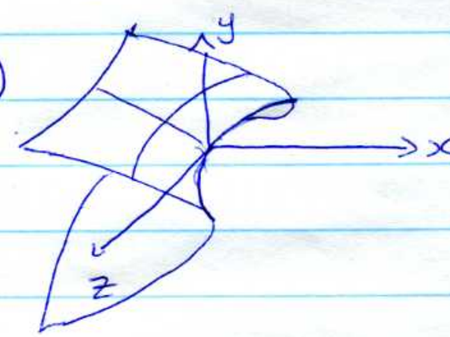
g)



12.6.5 $x=y^2-z^2$

This is a hyperbolic paraboloid.
Saddle point along the x-axis.

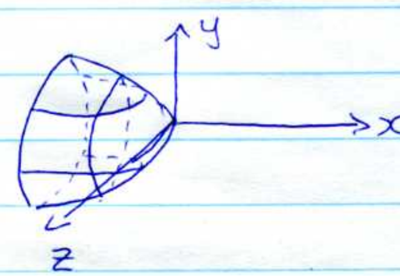
k)



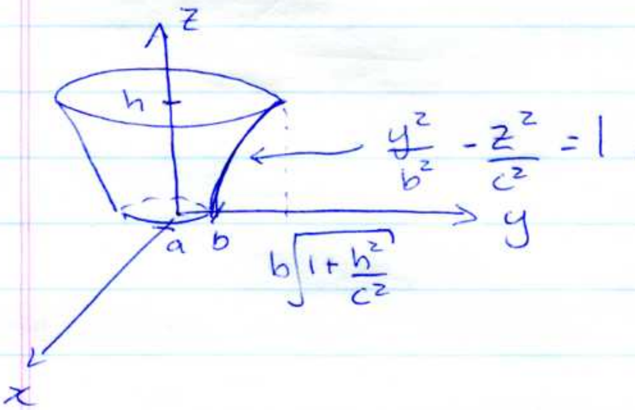
12.6.6 $x=-y^2-z^2$

This is an elliptical paraboloid.
opens along the negative x-axis.

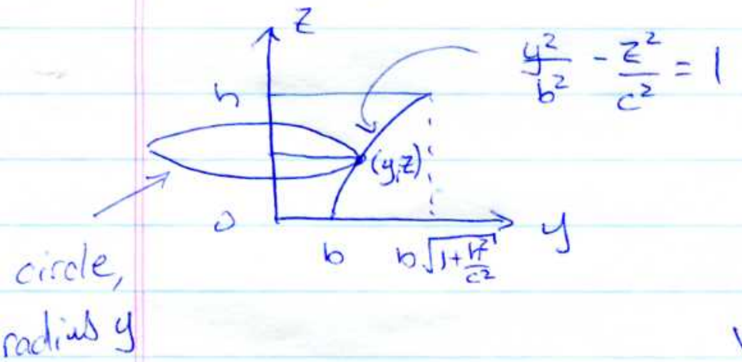
e)



12.6.8a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ between $z=0$ and $z=h$.



If we tried to do this as a rotation of a line in the yz-plane, we'd get



radius of circle $= y = b\sqrt{1+\frac{z^2}{c^2}}$

area of circle $= \pi r^2$

$= \pi b^2\left(1+\frac{z^2}{c^2}\right)$

$$\text{Volume} = \int_0^h \pi b^2\left(1+\frac{z^2}{c^2}\right)dz$$

$$= b^2 h\pi + \frac{b^2h^3\pi}{3c^2}.$$

Problem: This is the volume of

$\frac{x^2}{b^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ between $z=0$, $z=h$.

12.6.80a continued

We have an ellipse, not a circle. We can modify what we've done as follows:

$$\text{Volume} = \int_0^h (\text{area of ellipse})\, dz$$

where the ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 + \dfrac{z^2}{c^2}$

$$\Rightarrow \frac{x^2}{\left(a\sqrt{1+z^2/c^2}\right)^2} + \frac{y^2}{\left(b\sqrt{1+z^2/c^2}\right)^2} = 1$$

Area of this ellipse $= \pi a b \left(1 + \dfrac{z^2}{c^2}\right)$

$$\Rightarrow \text{Volume} = \int_0^h \pi a b \left(1 + \frac{z^2}{c^2}\right) dz$$

$$= \pi a b \left(h + \frac{h^3}{3c^2}\right)$$

If $a=b$, we recover what we had earlier.