Section 12.1 Three Dimensional Coordinate System

Problem (12.1.34) Write an inequality that describes the closed region bounded by the spheres of radius 1 and radius 2 centered at the origin.

A sphere of radius r centered at the origin is given by $x^2+y^2+z^2=r^2$. So the region we want is described by $x^2+y^2+z^2 \geq 1$ and $x^2+y^2+z^2 \leq 4$, which we can combine as

$$1 \leq x^2+y^2+z^2 \leq 4.$$

For a sketch see the associated *Mathematica* file.

Problem (12.1.52) Find the center and radius of the sphere $3x^2+3y^2+3z^2+2y-2z=9$.

We answer this question by completing the square in the three variables, and then writing the equation in the form $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$.

The variable x is already in the proper form.

$$\begin{aligned} 3y^2+2y &= 3\left(y^2+\frac{2}{3}y\right) \\ &= 3\left(y^2+\frac{2}{3}y+\frac{1}{9}-\frac{1}{9}\right) \\ &= 3\left(y+\frac{1}{3}\right)^2-\frac{1}{3} \end{aligned}$$

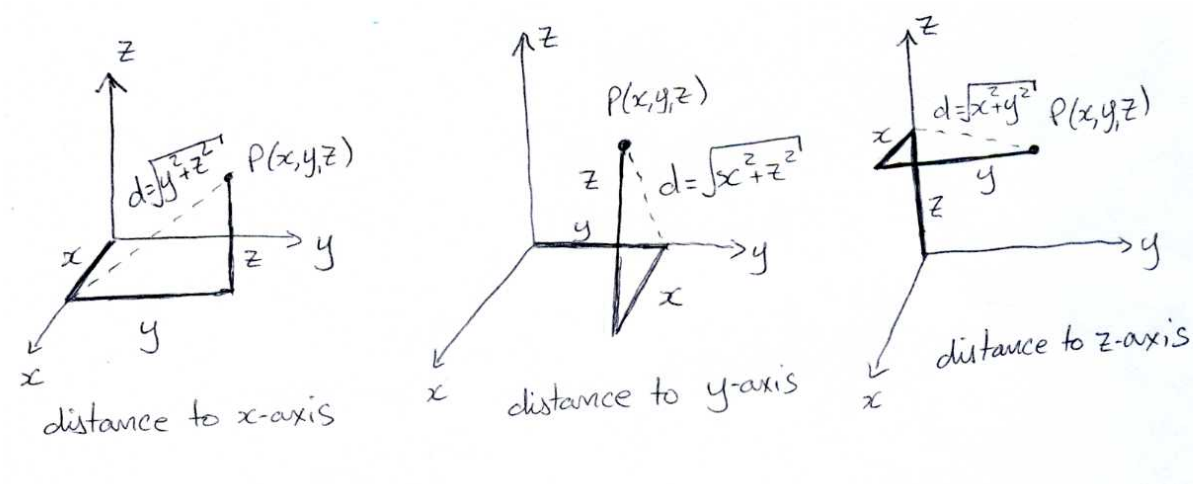
$$\begin{aligned} 3z^2-2z &= 3\left(y^2-\frac{2}{3}y\right) \\ &= 3\left(y^2+\frac{2}{3}y-\frac{1}{9}-\frac{1}{9}\right) \\ &= 3\left(y-\frac{1}{3}\right)^2-\frac{1}{3} \end{aligned}$$

$$\begin{aligned} 3x^2+3y^2+3z^2+2y-2z &= 9 \\ 3x^2+3\left(y+\frac{1}{3}\right)^2-\frac{1}{3}+3\left(y-\frac{1}{3}\right)^2-\frac{1}{3} &= 9 \\ x^2+\left(y+\frac{1}{3}\right)^2+\left(y-\frac{1}{3}\right)^2 &= 3+\frac{2}{9}=\frac{29}{9} \end{aligned}$$

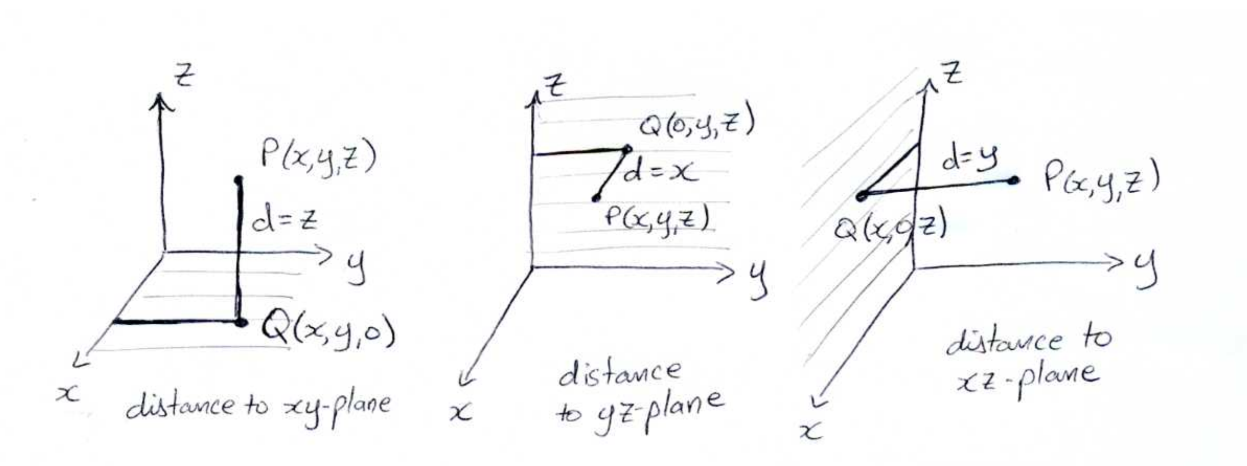
So this is a circle of radius $\sqrt{29}/3$ centered at $(0,-1/3,1/3)$.

Sketch in the *Mathematica* file.

Problem (12.1.53) Find a formula for the distance from the point $P(x, y, z)$ to the x-axis, the y-axis, and the z-axis.



Problem (12.1.54) Find a formula for the distance from the point $P(x, y, z)$ to the xy-plane, the yz-plane, and the xz-plane.



The distances are $|z|$, $|x|$, and $|y|$.