EX $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt = \int_{\pi/4}^{\pi/3} \frac{\sec\theta\tan\theta\,d\theta}{\sec^3\theta\tan\theta}$

All this is for the substitution!

Let $t = \sec\theta$

$dt = \sec\theta\tan\theta\,d\theta$

$t^3 = \sec^3\theta$

$\sqrt{t^2-1} = \sqrt{\sec^2\theta - 1}$

$= \tan\theta$

change limits: when $t=\sqrt{2}$, $\theta = \operatorname{arcsec}(\sqrt{2}) = \pi/4$

$t = 2$, $\theta = \operatorname{arcsec}(2)$

$= \arccos(\frac{1}{2}) = \pi/3$

$= \displaystyle\int_{\pi/4}^{\pi/3} \cos^2\theta\,d\theta$

$= \displaystyle\int_{\pi/4}^{\pi/3} \frac{1}{2}(1+\cos 2\theta)\,d\theta$

$= \displaystyle\frac{1}{2}\int_{\pi/4}^{\pi/3} d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/3} \cos 2\theta\,d\theta$

Another substitution

let $w = 2\theta$ when $\theta = \pi/4$, $w = \pi/2$

$dw = 2d\theta$ $\theta = \pi/3$, $w = 2\pi/3$

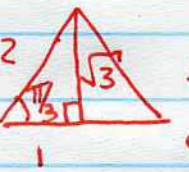
$= \displaystyle\frac{\theta}{2}\Big|_{\pi/4}^{\pi/3} + \frac{1}{4}\int_{\pi/2}^{2\pi/3} \cos w\,dw$

$= \displaystyle\frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) + \frac{1}{4}\sin w\Big|_{\pi/2}^{2\pi/3}$

$= \displaystyle\frac{\pi}{24} + \frac{1}{4}\left(\sin(2\pi/3) - \sin(\pi/2)\right)$

$\sin \pi/2 = 1.$

$\sin(2\pi/3) = 2\sin(\pi/3)\cos(\pi/3)$ (trig identity)



$\sin(\pi/3) = \frac{\sqrt{3}}{2}$ so $\sin\left(\frac{2\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\cos(\pi/3) = \frac{1}{2}$ $= \frac{\sqrt{3}}{2}$

Finally, $\displaystyle\int_{\sqrt{2}}^{2} \frac{dt}{t^3\sqrt{t^2-1}} = \frac{\pi}{24} + \frac{1}{4}\left(\frac{\sqrt{3}}{2} - 1\right)$