Try to do these without a calculator. Remember you can check your answers by multiplying out.

Common factors in terms

Usually you want the greatest common factor so you can work with smaller numbers:

$$48x^2 + 96x + 36 = 12(4x^2 + 8x + 3).$$

Factoring by Grouping

This is sort of like using the distribution property in the other direction:

$$(3y - 8)(2x - 7) = 3y(2x - 7) - 8(2x - 7) \quad \text{(distribution property)}$$
$$3y(2x - 7) - 8(2x - 7) = (3y - 8)(2x - 7) \quad \text{(factoring by grouping)}$$

Factoring trinomials of form $x^2 + bx + c$

$x^2 + bx + c = (x + m)(x + n)$ where m and n are two numbers whose product is c and sum is b.

Factoring trinomials of form $ax^2 + bx + c$

The Grouping Method to factor trinomials of form $ax^2 + bx + c$:

1. Determine the grouping number ac.
2. Find two numbers whose product is ac and sum is b.
3. Use these numbers to write bx as the sum of two terms.
4. Factor by grouping.
5. Check your answer by multiplying out.

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b).$$

Perfect Square (sum and difference)

$$a^2 + 2ab + b^2 = (a + b)^2,$$
$$a^2 - 2ab + b^2 = (a - b)^2.$$

Sum of Cubes, and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2),$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

Remember that for the cubes, you will not be able to factor the resulting quadratic using the techniques of this unit.

Questions

1. Factor $9x^2 + 9x + 2$.

2. Factor $4x^2 + 11x + 6$.

3. Factor $15x^2 - 34x + 15$.

4. Factor $3a^2 - 10a - 8$.

Note: My solutions to 1-4 contain both trial and error and grouping method (which is why they are so long–you don't need to do both).

5. Factor $12x^2 - 2x - 18x^3$.

6. Factor $4x^2 - 28x - 72$.

7. Factor $7x^2 + 3x - 2$.

8. Factor $14x^2 - x^3 + 32x$.

9. Factor $30x^3 - 25x^2y - 30xy^2$.

10. Factor $27x^4 - 64x$.

My solutions for 5-10 are brief, the minimum you would need to show to get the correct answer. I will leave it to your to check your answers by multiplying out.

Solutions

1. Factor $9x^2 + 9x + 2$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 9:	9 and 1
	3 and 3
Factors of 2:	1 and 2

Possible Factors	Middle Term	Correct?
$(9x + 1)(x + 2)$	$19x$	No
$(9x + 2)(x + 1)$	$11x$	No
$(3x + 1)(3x + 2)$	$9x$	Yes

Check: $(3x + 1)(3x + 2) = 9x^2 + 3x + 6x + 2 = 9x^2 + 9x + 2$.

Grouping Method

$9x^2 + 9x + 2$ has grouping number $9 \times 2 = 18$.

Find two numbers whose product is 18 and whose sum is 9: 3 and 6.

Now write the $9x$ term as two terms based on the numbers you found.

$$\begin{aligned} 9x^2 + 9x + 2 &= 9x^2 + 3x + 6x + 2 \\ &\quad \text{(red terms have a factor of } 3x) \\ &\quad \text{(blue terms have a factor of 2)} \\ &= 3x(3x + 1) + 2(3x + 1) \\ &\quad \text{(both terms have a factor of } 3x + 1) \\ &= (3x + 2)(3x + 1) \end{aligned}$$

Check: $(3x + 1)(3x + 2) = 9x^2 + 3x + 6x + 2 = 9x^2 + 9x + 2$.

You might also have written the following, which is entirely correct.

$9x^2 + 9x + 2 = 9x^2 + 6x + 3x + 2$
(red terms have a factor of $3x$)
(blue terms have no factor (it appears))
$= 3x(3x + 2) + (3x + 2)$
$= 3x(3x + 2) + 1(3x + 2)$ (those blue terms actually have a factor of 1, so put it in)
(both terms have a factor of $3x + 2$)
$= (3x + 1)(3x + 2)$

2. Factor $4x^2 + 11x + 6$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	4 and 1
	2 and 2
Factors of 6:	1 and 6
	2 and 3

Possible Factors	Middle Term	Correct?
$(4x + 1)(x + 6)$	$25x$	No
$(4x + 2)(x + 3)$	$14x$	No
$(2x + 1)(2x + 6)$	$14x$	No
$(2x + 2)(2x + 3)$	$10x$	No
$(4x + 6)(x + 1)$	$20x$	No
$(4x + 3)(x + 2)$	$11x$	Yes

Check: $(4x + 3)(x + 2) = 4x^2 + 3x + 8x + 6 = 4x^2 + 11x + 6$.

Grouping Method

$4x^2 + 11x + 6$ has grouping number $4 \times 6 = 24$.

Find two numbers whose product is 24 and whose sum is 11: 3 and 8.

Now write the $11x$ term as two terms based on the numbers you found.

$4x^2 + 11x + 6 = 4x^2 + 3x + 8x + 6$
(red terms have a factor of x)
(blue terms have a factor of 2)
$= x(4x + 3) + 2(4x + 3)$
(both terms have a factor of $4x + 3$)
$= (x + 2)(4x + 3)$

Check: $(4x + 3)(x + 2) = 4x^2 + 3x + 8x + 6 = 4x^2 + 11x + 6$.

3. Factor $15x^2 - 34x + 15$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 15:	15 and 1	Signs must be negative since the middle term is negative $-34x$.
	3 and 5	

Possible Factors	Middle Term	Correct?
$(15x-15)(1x-1)$	$-30x$	No
$(15x-3)(1x-5)$	$-78x$	No
$(3x-15)(5x-1)$	$-78x$	No
$(3x-3)(5x-5)$	$-30x$	No
$(15x-1)(1x-15)$	$-226x$	No
$(15x-5)(1x-3)$	$-50x$	No
$(3x-1)(5x-15)$	$-50x$	No
$(3x-5)(5x-3)$	$-34x$	Yes (finally!)

Check: $(3x-5)(5x-3) = 15x^2 - 25x - 9x + 15 = 15x^2 - 34x + 15$.

Grouping Method

$15x^2 - 34x + 15$ has grouping number $15 \times 15 = 225$.

Find two numbers whose product is 225 and whose sum is -34: -9 and -25.

Hint: Look for numbers "in the middle" rather than on the edges (this would help in the trial and error as well). What I mean is, don't start with $-1 \times (-225)$ since that does equal 225, but obviously won't have a sum of -34. This will just speed things up, you can always examine all the factors of 225.

Now write the $-34x$ term as two terms based on the numbers you found.

$$\begin{aligned}
15x^2 - 34x + 15 &= 15x^2 - 9x - 25x + 15 \\
&\quad \text{(red terms have a factor of } 3x) \\
&\quad \text{(blue terms have a factor of } 5) \\
&= 3x(5x-3) + 5(-5x+3) \\
&= 3x(5x-3) - 5(5x-3) \text{ (factor a } -1 \text{ out of second term to get common factor in each term)} \\
&\quad \text{(both terms have a factor of } 5x-3) \\
&= (3x-5)(5x-3)
\end{aligned}$$

Check: $(3x-5)(5x-3) = 15x^2 - 25x - 9x + 15 = 15x^2 - 34x + 15$.

4. Factor $3a^2 - 10a - 8$.

Since the coefficient of a^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 3:	3 and 1	
Factors of 8:	2 and 4	Signs must be opposite since the last term is negative (-8).
	1 and 8	

Possible Factors	Middle Term	Correct?
$(3a-2)(1a+4)$	$+10x$	No, but only out by sign, so switch them
$(3a+2)(1a-4)$	$-10x$	Yes

Check: $(3a+2)(a-4) = 3a^2 - 12a + 2s - 8 = 3a^2 - 10a - 8$.

Grouping Method

$3a^2 - 10a - 8$ has grouping number $3 \times (-8) = -24$.

Find two numbers whose product is -24 and whose sum is -10: -12 and 2.

Now write the $-10a$ term as two terms based on the numbers you found.

$$\begin{aligned}3a^2 - 10a - 8 &= 3a^2 - 12a + 2a - 8\\ &\text{(red terms have a factor of } 3a)\\ &\text{(blue terms have a factor of 2)}\\ &= 3a(a-4) + 2(a-4)\\ &\text{(both terms have a factor of } a-4)\\ &= (3a+2)(a-4)\end{aligned}$$

Check: $(3a+2)(a-4) = 3a^2 - 12a + 2s - 8 = 3a^2 - 10a - 8$.

5.

$$\begin{aligned}12x^2 - 2x - 18x^3 &= 2x(6x - 1 - 9x^2) \text{ Factor } 2x\\ &= -2x(9x^2 - 6x + 1) \text{ Reorder and factor } -1\\ &= -2x(3x-1)^2 \text{ Perfect square (difference), } 3x \text{ and } 1\end{aligned}$$

6.

$$\begin{aligned}4x^2 - 28x - 72 &= 4(x^2 - 7x - 18) \text{ Factor 4. Need two numbers: sum is } -7\text{, product is } -18\text{: } -9, 2\\ &= -2x(x-9)(x-2)\end{aligned}$$

7. $7x^2 + 3x - 2$ is a prime polynomial. You cannot find two integers whose sum is 3 and product is -14. However, this can be factored using the quadratic formula.

The solution to $7x^2 + 3x - 2 = 0$ is

$$\begin{aligned}x &= \frac{-3 \pm \sqrt{(3)^2 - 4(7)(-2)}}{2(7)}\\ &= \frac{-3 \pm \sqrt{65}}{14}\end{aligned}$$

We can use this to factor the original quadratic using the following logic.

If a quadratic has two roots, r_1 and r_2, then the quadratic must have factors $(x - r_1)$ and $(x - r_2)$.

If the quadratic has leading coefficient a, then the quadratic can be written as $a(x - r_1)(x - r_2)$.

In this case, we therefore have

$$7x^2 + 3x - 2 = 7\left(x - \frac{-3 + \sqrt{65}}{14}\right)\left(x - \frac{-3 - \sqrt{65}}{14}\right)$$

Factoring using the quadratic formula will be useful later on.

8.

$$\begin{aligned}14x^2 - x^3 + 32x &= -x(-14x + x^2 - 32) \text{ Factor } x.\\ &= -x(x^2 - 14x - 32) \text{ Reorder. Need two numbers: sum is } -14\text{, product is } -32\text{: } -16, 2\\ &= -x(x-16)(x+2)\end{aligned}$$

9.

$$\begin{aligned}30x^3 - 25x^2y - 30xy^2 &= 5x(6x^2 - 5xy - 6y^2) \text{ Factor } 5x\text{. Grouping Method is next, let } y \text{ follow along with constant.}\\ &= 5x(6x^2 - 5xy - 6y^2) \text{ Need two numbers: sum is } -5y\text{, product is } -36y^2\text{: } -9y, 4y\\ &= 5x\left[\underline{6x^2 - 9yx} + \underline{4yx - 6y^2}\right] \text{ find greatest common factor in first two terms and last two terms.}\\ &= 5x\left[3x(2x-3y) + 2y(2x-3y)\right]\\ &= 5x\left[(3x+2y)(2x-3y)\right] = 5x(3x+2y)(2x-3y)\end{aligned}$$

10.

$$\begin{aligned}27x^5 - 64x^2 &= x^2(27x^3 - 64) \text{ Factor } x^2\text{. Difference of cubes with } (3x)^3 = 27x^3 \text{ and } 4^3 = 64.\\ &= x^2(3x-4)((3x)^2 + (3x)(4) + 4^2)\\ &= x^2(3x-4)(9x^2 + 12x + 16)\end{aligned}$$