The test will have five types of problems:

- (10 Questions $\times$ 1 mark each = 10 marks total) Matching
- (10 $\times$ 1 mark = 10 marks) True or False
- (10 $\times$ 2 marks = 20 marks) Multiple Choice
- (6 $\times$ 5 marks = 30 marks) Short Answer
- (3 $\times$ 10 marks = 30 marks) Long Answer

Section 1. (10 $\times$ 1 mark = 10 marks) Matching Match term or quantity in left column to descriptions that best apply from the numbered columns.

Chinese Postman Problem _____

Hamiltonian circuit _____

spanning tree _____

critical path _____

complete graph _____

nearest neighbour algorithm _____

connected graph _____

list processing algorithm _____

vertex coloring _____

feasible region _____

1. a circuit on the graph that visits each vertex once and only once.
2. the shortest path in an order requirement digraph.
3. for every pair of its vertices there is at least one path connecting the two vertices.
4. the longest path in an order requirement digraph.
5. finding a circuit on a graph that covers every edge of the graph at least once and reuses the least number of edges (or if the edges have wights, has the circuit has the shortest length).
6. a heuristic algorithm for solving the traveling salesman problem (TSP) by determining a Hamiltonian circuit.
7. an algorithm that solves the minimum cost spanning tree problem.
8. a heuristic algorithm for assigning tasks to processors.
9. a heuristic algorithm for determining a good priority list from an order requirement digraph (ORD).
10. a circuit that covers each edge of a graph once, but not more than once.
11. an assignment of colors to the edges of a graph such that no two vertices joined by an edge have the same color.
12. a graph where every vertex is joined by an edge to every other vertex.
13. a subgraph of a connected graph that is a tree and includes all the vertices of the original graph.
14. the set of points which are possible solutions to a linear programming problem.
15. an assignment of colors to the vertices of a graph such that no two vertices joined by an edge have the same color.

Section 2. True or False Circle True (T) or False (F):

(1) The spanning tree of a graph must include all the edges of the graph . . . . T F

(2) If a graph has a Hamiltonian circuit given by ABCDEA (listing vertices) then DCBAED is also a Hamiltonian circuit on the graph . . . . T F

(3) The nearest neighbour algorithm for solving the travelling salesman problem always gives optimal results T F

(4) The nearest neighbour algorithm for solving the travelling salesman problem always produces the same result as the sorted edges algorithm . . . . T F

(5) The circuit produced by the nearest neighbour algorithm may depend on the starting city . . . . T F

(6) The circuit produced by the sorted edges algorithm may depend on the starting city . . . . T F

(7) An Euler circuit visits each vertex of a graph only once, but may not travel along all the edges of the graph . . . . T F

(8) In an order requirement digraph, the edges have a direction and a weight . . . . T F

(9) A graph which is not connected must have at least one vertex with valence zero . . . . T F

(10) The list processing algorithm for scheduling tasks always gives optimal results . . . . T F

(11) A vertex colouring of a graph means each edge connects vertices of the same colour. . . . . T F

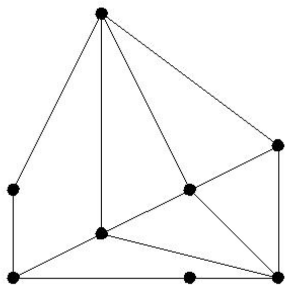
(12) A vertex colouring of a graph can be used to schedule final exams for students in a high school. . . . . T F

(13) Linear programming allows you to solve for the maximum of some quantity subject to certain constraints. . . . . T F

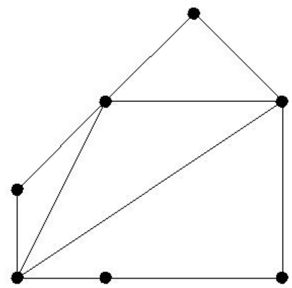
(14) In a linear programming problem, the feasible region represents the solutions which maximize some quantity. . . . . T F

Section 3. Multiple Choice Circle the most appropriate answer:

(1) Which of the following graphs have Euler circuits?



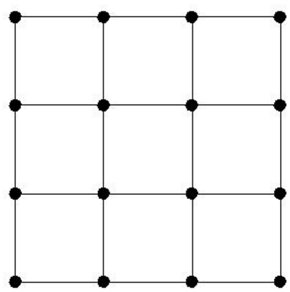
I



II

A) I only B) II only C) I and II both D) Neither I nor II

(2) In order to eulerize the graph below, what is the fewest number of edges that need to be duplicated?



A) 2 B) 3 C) 4 D) 6

(3) For which of the two situations below is it desirable to find an Euler circuit or an efficient eulerization of a graph?

I. A city employee must check the traffic signals at each intersection in downtown area to be certain they are working.

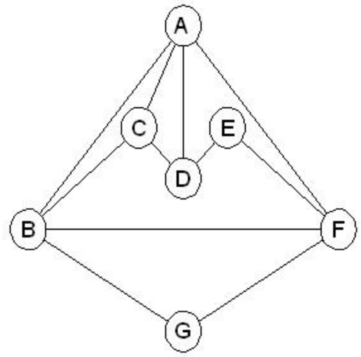
II. An employee of a power company reads the electric meters outside each house along the streets in a residential area.

A) I only B) II only C) I and II both D) Neither I nor II

(4) For the travelling salesman problem (Hamiltonian circuit) applied to four cities, how many <u>distinct</u> Hamiltonian tours are possible?

A) 3 B) 6 C) 12 D) 24

(5) Which of the following describes a Hamiltonian circuit for the following graph?

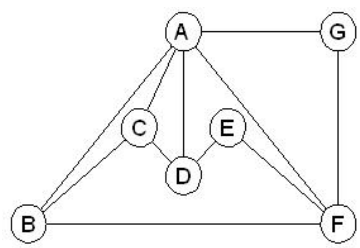


A) AFEDCBG B) ADEFGBA C) ABCDEFGBA D) ABGFEDCA

(6) A talent show producer needs to fit 17 acts of varying length into several segments. The segments will be no more than 45 minutes long, and are to be separated by intermissions. This problem could be solved by using

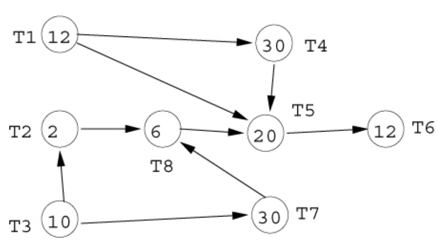
A) a Hamiltonian circuit.
B) linear programming.
C) the critical path scheduling algorithm.
D) none of these techniques would work to find a solution.

(7) Which of the following represents a spanning tree for the following graph?



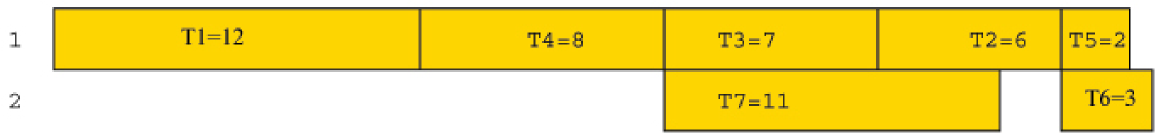
A) AG,GF,FB,AB B) AD,EF,DE,AG,DC,AB C) AD,EF,DE,AG,BC,AB D) Both B) & C)

(8) What is a critical path for the following order requirement digraph (with time given in minutes)?



A) T1,T4,T5,T6 B) T3,T2,T8,T5,T6 C) T3,T7,T8,T5,T6 D) Both B) & C)

(9) A company has a job scheduled on two processors in the following manner. To shorten the time needed to complete the job, the company plans to shorten the time spent on task T3. By how much can the time to complete T3 be reduced before there is no benefit in reducing it further?

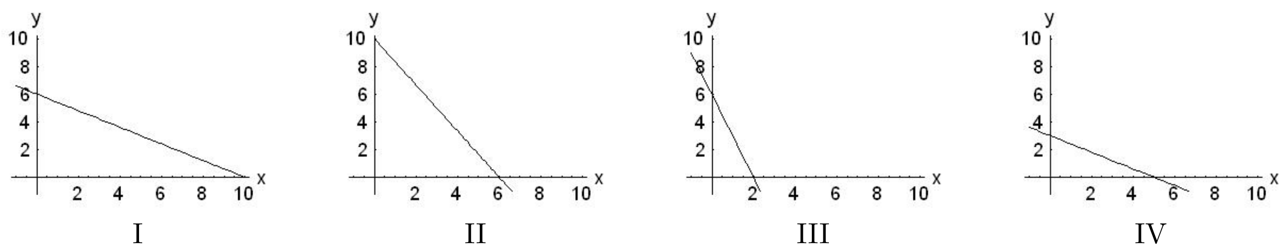


A) 0 B) 2 C) 4 D) 7

(10) A spanning tree on a graph H:

A) includes all the vertices on the graph H.
B) may contain a circuit.
C) is connected.
D) Both A) & C).

(11) The graph of the equation $3x + 5y = 30$ is



A) I B) II C) III D) IV

(12) The point of intersection of the lines whose equations are $2x + 3y = 12$ and $x + 5y = 13$ is

A) $(2, 3)$
B) $(3, 2)$
C) $(6, 0)$
D) $(-2, 3)$

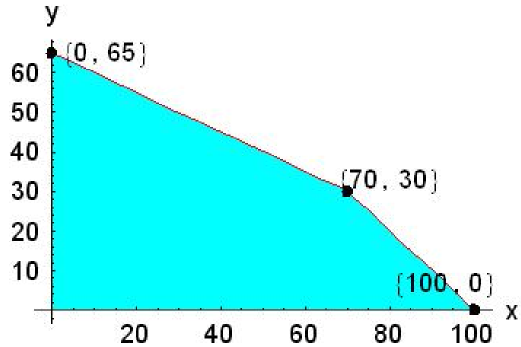
(13) Write the constraint equations for the following situation: A cheeseburger requires 5 oz of meat and 0.7 ounces of cheese while a superburger requires 7oz of meat and 0.6 oz of cheese. The burger stand has 350 oz of meat and 42 oz of cheese available. The profit on a cheeseburger is 10 cents and the profit on a superburger is 40 cents.

A) $5x + 7y \leq 350$, $0.7x + 0.6y \leq 42$, $x \geq 0$, $y \geq 0$.
B) $5x + 0.7y \leq 350$, $7x + 0.6y \leq 42$, $x \geq 0$, $y \geq 0$.
C) $5x + 7y \leq 10$, $0.7x + 0.6y \leq 42$, $x \geq 0$, $y \geq 0$.
D) $10x + 4y \leq 350$, $0.7x + 0.6y \leq 42$, $x \geq 0$, $y \geq 0$.

(14) Suppose in a linear programming problem you find the feasible region has corner points of $(0, 0)$, $(5, 0)$, $(0, 4)$ and $(2, 3)$. If the profit formula is $P = \$3x + \$4y$, then the maximum profit is

A) \$12
B) \$13
C) \$14
D) \$18

Problems (15)–(17) are related the the following sketch of a feasible region:



(15) What is the maximum profit when $P = \$3x + \$4y$?

A) \$260 B) \$330 C) \$300 D) \$370

(16) What values of x and y produce the maximum profit when $P = \$4x + \$4y$?

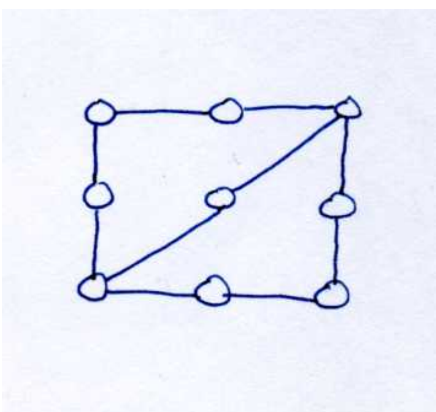
A) only $x = 0$, $y = 65$ B) only $x = 70$, $y = 30$ C) only $x = 100$, $y = 0$ D) only $x = 100$, $y = 0$ and $x = 70$, $y = 30$ E) none of the above

(17) Which point is not in the feasible region?

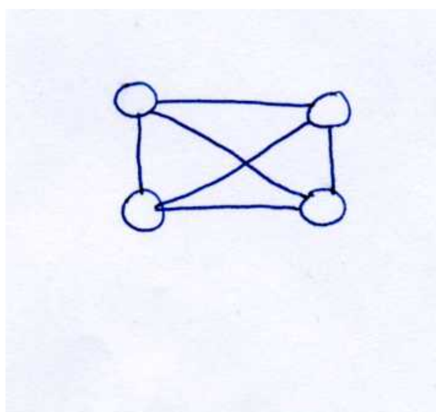
A) $(0, 0)$ B) $(60, 30)$ C) $(30, 60)$ D) $(70, 30)$

Section 4. Short Answer

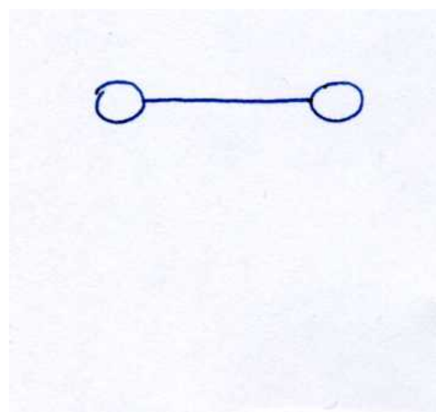
(1) Find the most efficient Eulerization of the following graphs:



(a)



(b)



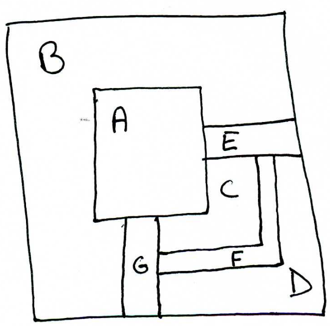
(c)

(2) The following table represents scheduling conflicts between individuals.

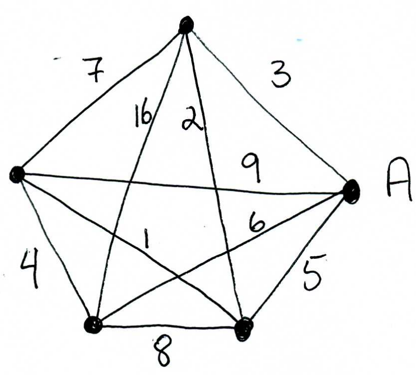
	A	B	C	D
A		X	X	
B	X			X
C	X			
D		X		

Construct a graph that conveys this information. What is the chromatic number of the graph you created?

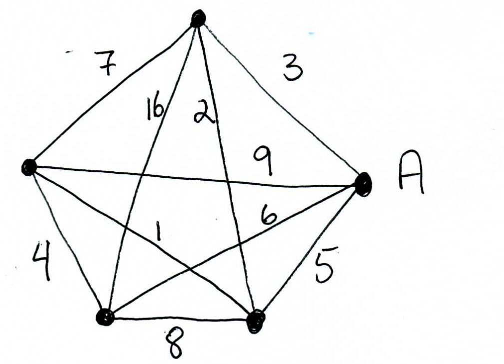
(3) Create a graph that can used to help you color the following map. Then, determine the chromatic number of the graph you created.



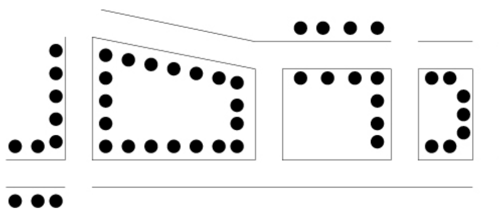
(4) For the following graph, use the nearest neighbour algorithm starting at vertex A to determine a Hamiltonian circuit. You can simply highlight the included edges on the graph.



(5) For the following graph, use Kruskal's algorithm to determine a minimum cost spanning tree. You can simply highlight the included edges on the graph.



(6) For following street network, the dots represent parking meters which much be checked by a patrol officer on foot. Draw a graph beside the network which would be useful to help them find an efficient route.

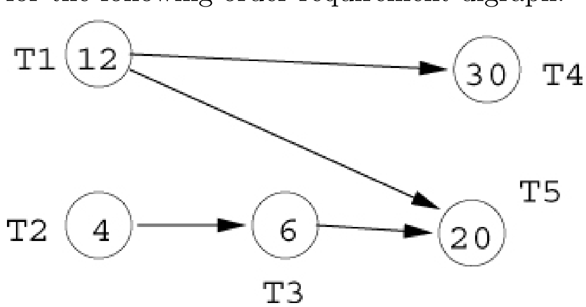


(7) Use the decreasing version of the First Fit Algorithm to schedule advertisements of lengths 30,40,15,20,16,45,50,20 seconds in 80 second slots for a radio program.

First Fit Algorithm Put items into the first bin until the next item doesn't fit. Then open a new bin, but leave the first bin open. Put each item from the list into the lowest numbered bin in which it will fit, opening new bins when an item won't fit in any of the open bins.

Section 5. Long Answer

(1) Use critical path scheduling and then the list processing algorithm to construct a schedule on two processors for the following order requirement digraph.



(2) Sketch the feasible region for the following linear programming mixture problem, and then determine how many vases and bowls should be produced to maximize profit. A mixture chart is not required for your solution, although it may help you solve the problem.

Kim and Lynn produce pottery vases and bowls. A vase requires 25 oz of clay and 5 oz of glaze. A bowl requires 20 oz of clay and 10 oz of glaze. There are 500 oz of clay and 160 oz of glaze available. The profit on one vase is \$5 and the profit on one bowl is \$3.

(3) When two towns are within 145 miles of each other, the frequency used by a certain type of emergency response system for the towns requires that they be on different frequencies to avoid interference with each other.

The following table shows mileage distance between six towns.

	A	B	C	D	E	F
A		290	277	168	303	133
B	290		132	83	79	201
C	277	132		153	58	164
D	168	83	153		140	71
E	303	79	58	140		196
F	133	201	164	71	196	

1. What are minimum number of frequencies that are needed for each town to have its emergency broadcasts not interfere with neighbouring towns?
2. How many different towns would be assigned to each frequency used?

(4) For the following order requirement digraph and priority list $T_1, T_5, T_6, T_2, T_4, T_3, T_8, T_9, T_7, T_{10}$, determine a schedule on three processors using the list processing algorithm.