Composition of functions

Given two functions f and g, the **composite** function $f \circ g$ (called the composition of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

and the domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$ values in the domain of f.

Composition example If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4 - x^2}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and give the domains.

Solution

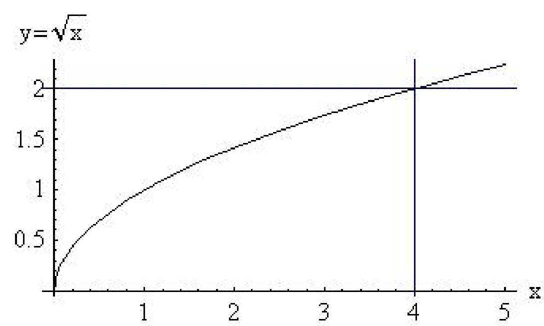
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{4 - x^2}) \\ &= \sqrt{(\sqrt{4 - x^2})} \\ &= (4 - x^2)^{1/4} \end{aligned}$$

The domain of g is $[-2, 2]$, the range of g is $[0, 2]$. The domain of f is $[0, \infty)$. Since the range of g is contained in the domain of f, we can draw values from the entire domain of g and get real valued output when we act on $g(x)$ with f. The domain of $f \circ g$ is therefore $[-2, 2]$.

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= \sqrt{4 - (\sqrt{x})^2} \\ &= \sqrt{4 - x} \end{aligned}$$

The domain of f is $[0, \infty)$, and the range of f is $[0, \infty)$. Now, the domain of g is $[-2, 2]$ so we can only use values of x which, when acted on by f, land in the set $[-2, 2]$. Otherwise, we can't act on $f(x)$ with g and get a real value out.

Here is a sketch to help us find which values in the domain of f lie in the set $[-2, 2]$.



We can see that we can only draw from $x \in [0, 4]$. The domain of $g \circ f$ is $[0, 4]$.