

Example 7.1.16 Solve the system algebraically.

$$\begin{aligned}y &= x^3 + x^2 \\y &= -x^2\end{aligned}$$

Use the method of substitution. Substitute the first equation into the second and solve for x :

$$\begin{aligned}y &= -x^2 \\x^3 + x^2 &= -x^2 \\x^3 + 2x^2 &= 0 \\x^2(x + 2) &= 0\end{aligned}$$

The solution to the above is $x^2 = 0 \rightarrow x = 0$ and $x + 2 = 0 \rightarrow x = -2$.

We can use the second original equation to determine y for each x value we found:

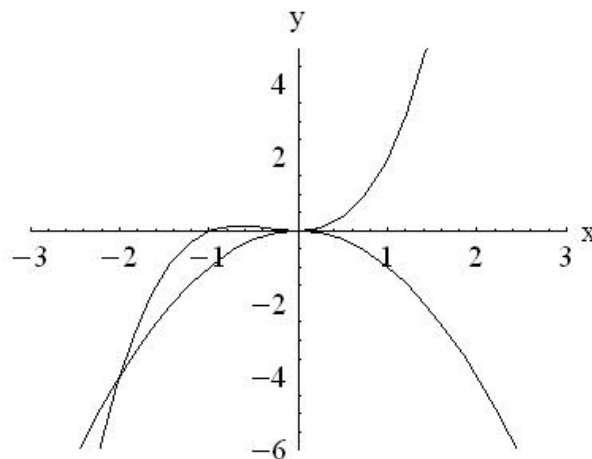
$$\begin{aligned}y &= -x^2 \\&= -(0)^2 = 0\end{aligned}$$

So one solution is $(x, y) = (0, 0)$.

$$\begin{aligned}y &= -x^2 \\&= -(-2)^2 = -4\end{aligned}$$

So another solution is $(x, y) = (-2, -4)$.

There are two solutions, $(x, y) = (0, 0)$ and $(x, y) = (-2, -4)$. The sketch below confirms this.



Example 7.1.50 Find the dimensions of a rectangular cornfield with a perimeter of 220 yd and an area of 3000 yd².

We need to introduce some notation to help us solve this problem. Let us say that the cornfield has length x and width y (we are told it is rectangular).

Therefore, the perimeter of the cornfield is $2x + 2y = 220$ and the area of the cornfield is $xy = 3000$.

Thus, we must solve the system of equations

$$\begin{aligned} 2x + 2y &= 220 \\ xy &= 3000 \end{aligned}$$

We can do this using the method of substitution.

Solve the first equation for y :

$$\begin{aligned} 2x + 2y &= 220 \\ x + y &= 110 \\ y &= 110 - x \end{aligned}$$

Now, substitute this into the second equation, and solve for x :

$$\begin{aligned} xy &= 3000 \\ x(110 - x) &= 3000 \\ 110x - x^2 &= 3000 \\ x^2 - 110x + 3000 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{110 \pm \sqrt{(110)^2 - 4(1)(3000)}}{2(1)} \\ &= \frac{110 \pm \sqrt{12100 - 12000}}{2} = \frac{110 \pm \sqrt{100}}{2} = \frac{110 \pm 10}{2} = 55 \pm 5 = 50 \text{ or } 60 \end{aligned}$$

We can use the second original equation to determine y for each x value we found:

$$\begin{aligned} xy &= 3000 \\ y &= \frac{3000}{x} = \frac{3000}{60} = 50 \end{aligned}$$

So one solution is $x = 60$, $y = 50$.

$$y = \frac{3000}{x} = \frac{3000}{50} = 60$$

Another solution is $x = 50$, $y = 60$.

Since x and y are the length and width of the rectangular cornfield, and it doesn't matter which is which, we see that there is only one solution to the problem. The cornfield is 50 yd by 60 yd.