Example 5.1.2 Evaluate without using a calculator; use identities rather than reference triangles. Find $\sec\theta$ and $\csc\theta$ if $\tan\theta = 3$ and $\cos\theta > 0$.

First, we need to figure out which Quadrant θ lies in:
$\tan\theta > 0$ means we are in Quadrant I or III.
$\cos\theta > 0$ means we are in Quadrant I or IV.
Therefore, we are in Quadrant I.

The $\sec\theta > 0$ and $\csc\theta > 0$ in Quadrant I.

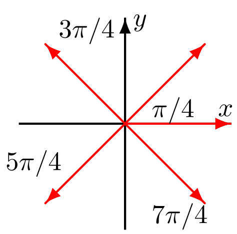
$$\begin{aligned}
\sec\theta &= \sqrt{1+\tan^2\theta} \quad (\text{choose } +\sqrt{\ } \text{ since } \sec\theta > 0)\\
&= \sqrt{1+3^2}\\
&= \sqrt{1+9} = \sqrt{10}
\end{aligned}$$

$$\begin{aligned}
\csc\theta &= \sqrt{\cot^2\theta+1} \quad (\text{choose } +\sqrt{\ } \text{ since } \csc\theta > 0)\\
&= \sqrt{\frac{1}{\tan^2\theta}+1}\\
&= \sqrt{\frac{1}{3^2}+1}\\
&= \sqrt{\frac{1}{9}+1}\\
&= \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}
\end{aligned}$$

Example 5.1.56 Find all solutions to the equation $2\sin^2 x = 1$ in the interval $[0, 2\pi)$.

$$\begin{aligned}
2\sin^2 x &= 1\\
\sin^2 x &= \frac{1}{2}\\
\sin x &= \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}
\end{aligned}$$

The angles around the unit circle corresponding to points with y coordinate equal to $+\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ are the following:



The equation has solution $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in the interval $[0, 2\pi)$.