Example 4.5.10 Describe the graph of the function in terms of basic trigonometric functions. Locate the vertical asymptotes and sketch two periods of the function.

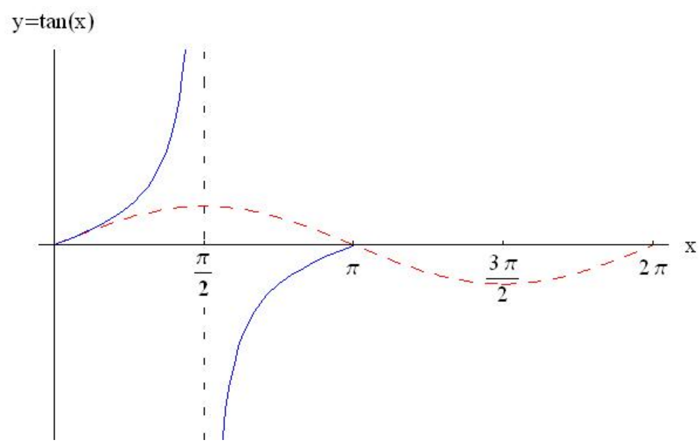
$$y = 3\tan(x/2)$$

The basic trig function is $y = \tan x$. It has period π. Therefore, this new function will complete one period when

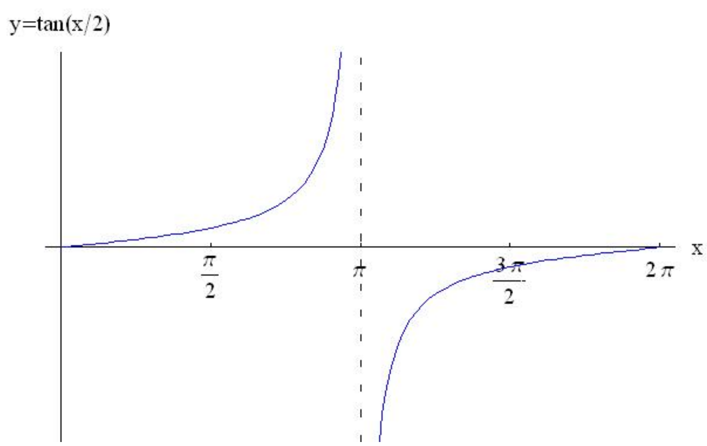
$$\begin{aligned} 0 \leq \quad x/2 \quad &\leq \pi \\ 0 \leq \quad x \quad &\leq 2\pi \end{aligned}$$

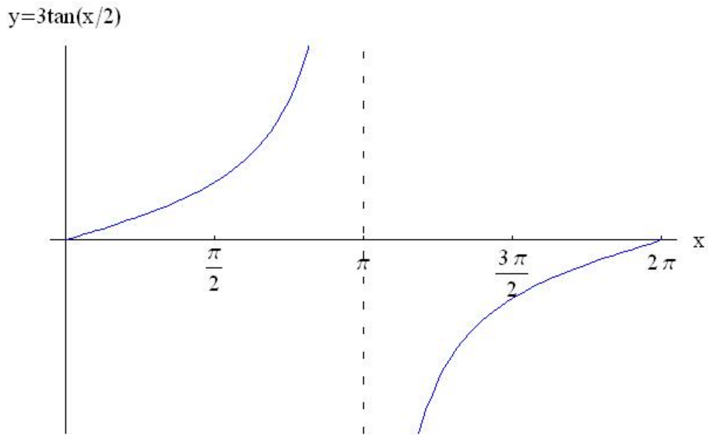
It will also have vertical stretch of 3 units compared to $y = \tan x$.

We construct the sketch of $\tan x$ by remembering how it relates to the sine function. The sine function is included in the sketch to help us.



Notice that we only drew one period of the sine and tangent function in the above sketch. Now we can do the vertical stretch and the horizontal stretch:



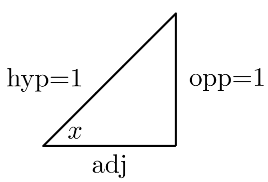


We see that the vertical asymptotes of $y = 3\tan(x/2)$ occur at $x = k\pi$, where $k = 0, \pm 1, \pm 2, \pm 3, \ldots$.

Example 4.5.33 Solve the equation $\csc x = 1$ in the interval $2\pi \le x \le 5\pi/2$. You should not need a calculator to solve this problem.

Let's construct a reference triangle to help us solve this problem. Since we are told not to use a calculator, we expect that the reference triangle will be one of the two special triangles, a 45-45-90 or a 30-60-90.

$$\csc x = 1 = \frac{1}{1} = \frac{\text{hyp}}{\text{opp}}$$



The adjacent side has length zero! This isn't one of our special triangles, it is a quadrantal angle! So we can still solve this, but it isn't one of our special triangles after all.

The cosecant equal to one means the sine is equal to one. The angle with sine equal to one is $\pi/2$, so $x = \pi/2$. However, we know that we can add (or subtract) 2π to this angle and get another solution to the equation. So $x = \pi/2 + 2\pi = 5\pi/2$ is also a solution. Since this is in the interval we want, this is our solution. Here is a sketch of the cosecant function so you can see the solution