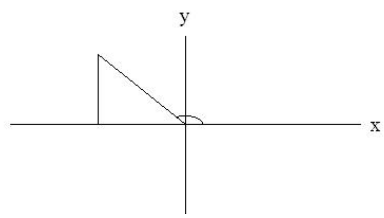
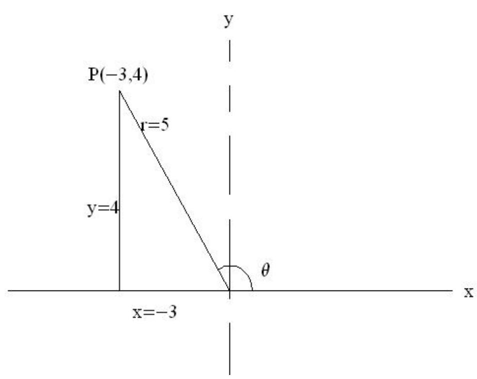
Example 4.3.48 Find $\csc\theta$ and $\cot\theta$ if $\tan\theta = -\dfrac{4}{3}$ and $\sin\theta > 0$.

Since the sine of θ is greater than zero, and the tangent is also less than zero, we know that we must have an angle that has a terminal side in the second quadrant.



Labelling, being careful to indicate quantities which are less than zero based on the quadrant the point P is in, yields:



where we have found r using the Pythagorean theorem.

$$\begin{aligned} x &= -3 \\ y &= 4 \\ r &= \sqrt{x^2+y^2} = \sqrt{(-3)^2+(4)^2} = \sqrt{25} = 5 \end{aligned}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{-3}{4} = -\frac{3}{4}$$

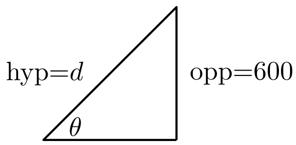
$$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{5}{4}$$

Example 4.3.59 An airplane flying at an altitude of 8000 ft passes directly over a group of hikers who are at 7400 ft. If θ is the angle of elevation from the hikers to the aircraft, find the distance d from the group to the aircraft for the given angle.

(a) $\theta = 45^{\circ}$ (b) $\theta = 90^{\circ}$ (c) $\theta = 140^{\circ}$

The angle of elevation is the angle through which the eye moves up from the horizontal to look at an object in the sky (if you have to look down, it is called the angle of depression).

The aircraft is 600 ft above the hikers when it passes directly overhead. Assuming that the aircraft's altitude does not change, this means that we have a reference triangle for the situation given by:



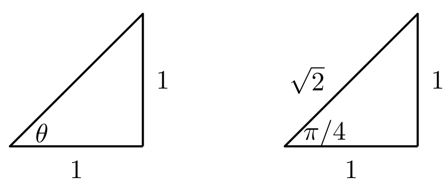
From the reference triangle, we have

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{600}{d}.$$

Therefore, $d = \dfrac{600}{\sin\theta}$. Now we have to evaluate the sine of the angle. I am including all of the details here to remind myself of how we get the sine for special angles.

(a) $\theta = 45^{\text{o}} = \dfrac{\pi}{4}$ radians:

Consider the isosceles (a triangle with two equal sides) right triangle given below.



The angle here must be $\pi/4$ radians, since this triangle is half of a square of side length 1.

$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

Therefore, $d = \dfrac{600}{\sin(\pi/4)} = \dfrac{600}{\left(\frac{1}{\sqrt{2}}\right)} = 600\sqrt{2} \sim 848.528$ ft.

(b) $\theta = 90^{\text{o}} = \dfrac{\pi}{2}$ radians:

If we think of the unit circle, we realize that the sine of $\pi/2$ is 1, since a rotation of $\pi/2$ puts us at the point $(0, 1)$, and the sine is the y value of the points on the unit circle.

Therefore, $d = \dfrac{600}{\sin(\pi/2)} = \dfrac{600}{(1)} = 600$ ft. This is when the aircraft is directly overhead.

(c) $\theta = 140^{\text{o}} = \dfrac{7\pi}{9}$ radians:

This is one of the angles we have to use a calculator to figure out. Therefore, $d = \dfrac{600}{\sin(7\pi/9)} \sim 933.434$ ft.