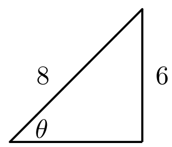
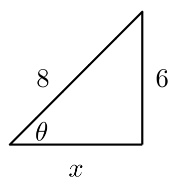
Example 4.2.6 Find the value of all six of the trigonometric functions of the angle θ given the following right angle triangle. Note the triangle is not drawn to scale.



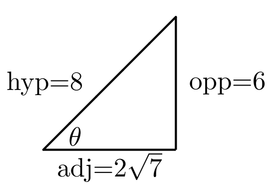
First, we need to use the Pythagorean theorem to find the length of the side adjacent to θ in the triangle, which I have labelled x:



$$\begin{aligned} x^2 + 6^2 &= 8^2 \\ x^2 &= 64 - 36 = 28 \\ x &= \sqrt{28} = 2\sqrt{7} \end{aligned}$$

We choose the positive root, not the negative root, since the angle is acute.

The triangle can be labelled as

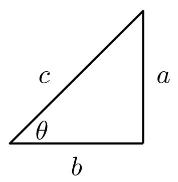


We need the trigonometric relations for the six trig functions:

$$\begin{aligned} \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{8} = \frac{3}{4} \\ \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4} \\ \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{2\sqrt{7}} = \frac{3}{\sqrt{7}} \\ \csc\theta &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{8}{6} = \frac{4}{3} \\ \sec\theta &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{8}{2\sqrt{7}} = \frac{4}{\sqrt{7}} \\ \cot\theta &= \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{7}}{6} = \frac{\sqrt{7}}{3} \end{aligned}$$

Example 4.2.77 Using the labeling of the triangle below, prove that if θ is an acute angle in any right triangle, then

$$(\sin\theta)^2 + (\cos\theta)^2 = 1.$$



We need to work out sine and cosine of the angle θ:

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}\\ \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}\end{aligned}$$

Now we need to show the following quantity reduces to 1 (break out the algebra!):

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= (\sin\theta)^2 + (\cos\theta)^2\\ &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2\\ &= \frac{a^2}{c^2} + \frac{b^2}{c^2}\\ &= \frac{a^2+b^2}{c^2}\\ &= \frac{c^2}{c^2} \text{ (by Pythagorean theorem)}\\ &= 1\end{aligned}$$