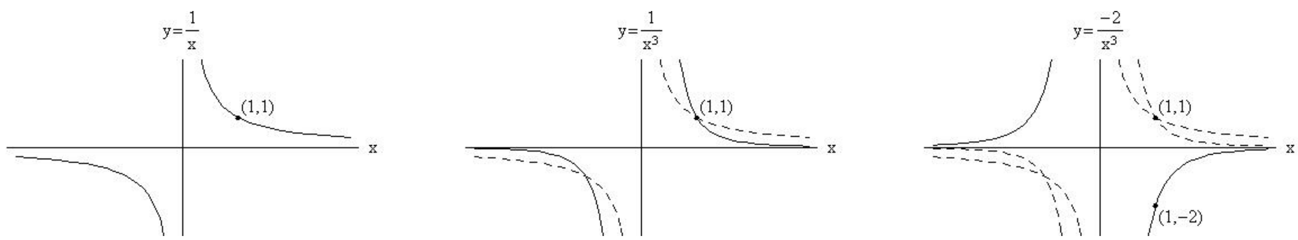
Example 2.2.30 State the power and constant of variation for the function $f(x) = -2x^{-3}$. Graph the function and analyze it.

The function has power -3. Since the power is less than zero, this is an inverse variation function. The constant of variation is -2.

Sketch (from the sketch of the basic function $y = x^{-1}$, so sketched by hand, not with a computer, although I used a computer to draw the sketches)



Domain: $x \in (-\infty, 0) \cup (0, \infty)$.
Range: $y \in (-\infty, 0) \cup (0, \infty)$.
Continuous: the function is discontinuous at $x = 0$.
Increasing/Decreasing: The function is increasing for $x \in (-\infty, 0)$, and increasing for $x \in (0, \infty)$.
Symmetric: The function is odd ($f(-x) = f(x)$).
Boundedness: The function is not bounded above or below.
Extrema: none.
Asymptotes: The function has a horizontal asymptote of $y = 0$, and a vertical asymptote at $x = 0$.
End Behaviour: $\lim_{x \to -\infty} (-2x^{-3}) = 0$ and $\lim_{x \to \infty} (-2x^{-3}) = 0$

Example 2.2.54 The power P (in watts) produced by a windmill is proportional to the cube of the wind speed v (in mph). If a wind of 10 mph generates 15 watts of power, how much power is generated by winds of 20, 40, and 80 mph? Make a table and explain the pattern.

We are told the power is proportional to the cube of the wind speed. Converting that to mathematics leads us to write

$$P \propto v^3$$

where P is the power in watts and v is the wind speed in mph. The symbol between them indicates that they are proportional to each other.

We need to know a relation for when they are equal, and we can get that by inserting a *proportionality constant* k:

$$P = kv^3$$

You could have started your solution with this relation. We don't know the value of k yet, but we can find it using some of the information given.

We are told that when $v = 10$ mph, $P = 15$ watts, and we can use this do determine the constant k:

$$P = kv^3$$

$$\begin{aligned} 15 &= k10^3 \\ \frac{15}{1000} &= k \\ k &= \frac{3}{200} \end{aligned}$$

and we can write the relation between wind speed in mph and power in watts as:

$$P = \frac{3}{200}v^3.$$

Now we can construct the table that is asked for:

v (mph)	$P = \frac{3}{200}v^3$ (watts)
10	15 (this data was given)
20	120
40	960
80	7680

The values for P at wind speeds of 20, 40, 80 mph were calculated using the relation. As the wind speed increases, the power grows rapidly, by a cubic relation. The proportionality constant $k = 3/200$ scales the cubic growth.