

Example 2.1.31 Find the vertex and axis of symmetry of $f(x) = 5x^2 - 6x + 4$.

$$\begin{aligned}
 f(x) &= 5x^2 - 6x + 4 \\
 &= 5\left(x^2 + \left(\frac{-6}{5}\right)x\right) + 4 \\
 &= 5\left(x^2 + \left(\frac{-6}{5}\right)x + \left(\frac{-3}{5}\right)^2 - \left(\frac{-3}{5}\right)^2\right) + 4 \\
 &= 5\left(x^2 + \left(\frac{-6}{5}\right)x + \left(\frac{-3}{5}\right)^2 - \left(\frac{-3}{5}\right)^2\right) + 4 \\
 &= 5\left[\left[x + \left(\frac{-3}{5}\right)\right]^2 - \left(\frac{-3}{5}\right)^2\right] + 4 \\
 &= 5\left[\left[x - \frac{3}{5}\right]^2 - \left(\frac{-3}{5}\right)^2\right] + 4 && \text{notice how our treatment of the minus sign for } b \text{ was done!} \\
 &= 5\left[x - \frac{3}{5}\right]^2 - 5\left(\frac{-3}{5}\right)^2 + 4 && \text{simplify} \\
 &= 5\left[x - \frac{3}{5}\right]^2 - \frac{9}{5} + \frac{20}{5} \\
 &= 5\left[x - \frac{3}{5}\right]^2 + \frac{11}{5}
 \end{aligned}$$

This is now in the vertex form, and we can identify the vertex as $(h, k) = \left(\frac{3}{5}, \frac{11}{5}\right)$ and axis of symmetry $x = \frac{3}{5}$.

Example 2.1.38 Use completing the square to describe the graph of $f(x) = 5x^2 - 25x + 12$.

$$\begin{aligned}
 f(x) &= 5x^2 - 25x + 12 \\
 &= 5(x^2 + (-5)x) + 12 \\
 &= 5\left(x^2 + (-5)x + \left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right)^2\right) + 12 \\
 &= 5\left(x^2 + (-5)x + \left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right)^2\right) + 12 \\
 &= 5\left[\left[x + \left(\frac{-5}{2}\right)\right]^2 - \left(\frac{-5}{2}\right)^2\right] + 12 \\
 &= 5\left[\left[x - \frac{5}{2}\right]^2 - \left(\frac{-5}{2}\right)^2\right] + 12 && \text{notice how our treatment of the minus sign for } b \text{ was done!} \\
 &= 5\left[x - \frac{5}{2}\right]^2 - 5\left(\frac{-5}{2}\right)^2 + 12 && \text{simplify} \\
 &= 5\left[x - \frac{5}{2}\right]^2 - \frac{125}{4} + \frac{48}{4} \\
 &= 5\left[x - \frac{5}{2}\right]^2 - \frac{77}{4}
 \end{aligned}$$

This is now in the vertex form, and we can identify the vertex as $(h, k) = \left(\frac{5}{2}, -\frac{77}{4}\right)$ and axis of symmetry $x = \frac{5}{2}$. It will open upwards since the coefficient in front of x^2 is positive. Since the vertex is below the x axis and the graph opens up, the graph will intersect the x -axis in two places. These roots can be found by setting $g(x) = 0$:

$$\begin{aligned}g(x) = 0 &= 5\left[x - \frac{5}{2}\right]^2 - \frac{77}{4} \\5\left[x - \frac{5}{2}\right]^2 &= \frac{77}{4} \\ \left[x - \frac{5}{2}\right]^2 &= \frac{77}{20} \\x - \frac{5}{2} &= \pm\sqrt{\frac{77}{20}} \\x &= \frac{5}{2} \pm \sqrt{\frac{77}{20}}\end{aligned}$$