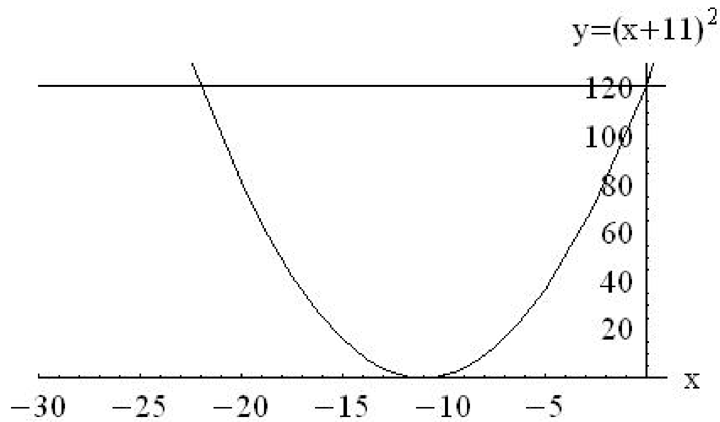
Example 1.1.30 Solve the equation $(x+11)^2 = 121$ algebraically and graphically.

Here is the algebraic solution:

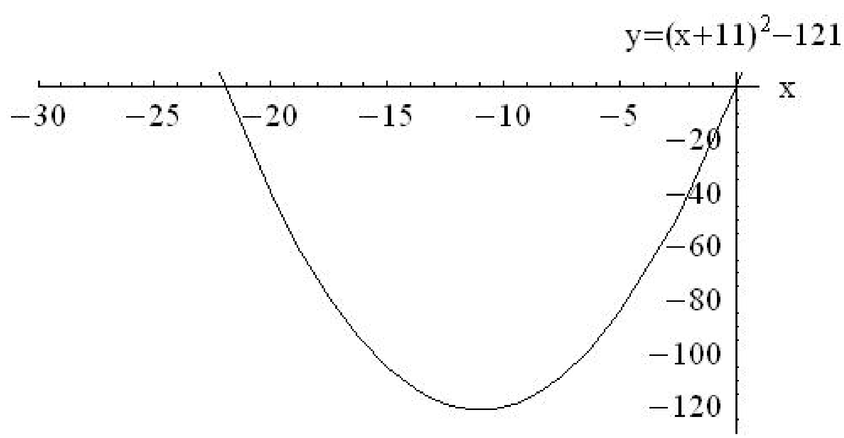
$$\begin{aligned} (x+11)^2 &= 121 \\ \sqrt{(x+11)^2} &= \pm\sqrt{121} \\ x+11 &= \pm 11 \\ x &= -11 \pm 11 \\ x = -11+11 \quad &\text{or} \quad x = -11-11 \\ x = 0 \quad &\text{or} \quad x = -22 \end{aligned}$$

We can also represent the solution graphically by sketching functions. Here are two possibilities. The sketches were generated using *Mathematica*, but you could use a calculator if you like.



From the sketch, we see the points of intersection of $y = (x+11)^2$ and $y = 121$ occur when $x = 0, -22$.

We could also sketch the following:



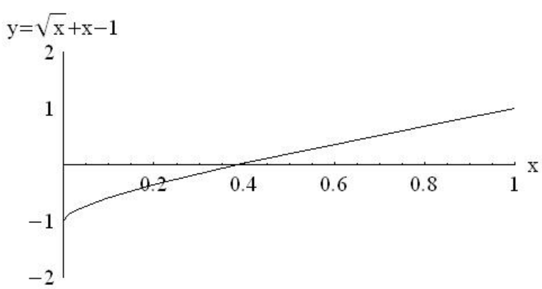
and identify the solution by looking for where $y = (x+11)^2 - 121 = 0$, which is where the function crosses the x-axis. In this sketch it is easier to see that one of the solutions is really $x = -22$ than in the previous sketch.

Example 1.1.38 Solve the equation $\sqrt{x}+x=1$ algebraically and graphically.

Here is the algebraic solution. We want to get rid of the square root, so we first isolate it and square both sides of the equation.

$$\begin{aligned}
\sqrt{x}+x &= 1\\
\sqrt{x} &= 1-x\\
(\sqrt{x})^2 &= (1-x)^2\\
x &= 1-2x+x^2\\
0 &= 1-3x+x^2\\
x^2-3x+1 &= 0 \text{ quadratic; use the quadratic formula}\\
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&= \frac{-(-3)\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}\\
&= \frac{3\pm\sqrt{9-4}}{2}\\
&= \frac{3\pm\sqrt{5}}{2}\\
x=\frac{3+\sqrt{5}}{2} &\quad \text{or} \quad x=\frac{3-\sqrt{5}}{2}
\end{aligned}$$

Now, we have to be a bit careful. Let's look at the sketch and see if our two solutions make sense.



From the sketch, we see the there is only one solution! It appears to be around 0.4, which corresponds to the solution $x=\frac{3-\sqrt{5}}{2}$ we found above.

The other algebraic solution doesn't appear to be a solution based on our graph. In fact, by squaring the equation in our algebraic solution (the blue line in the algebraic solution) we introduced an *extraneous solution*, which means it is a solution that arose do to our algebraic manipulations, but isn't a solution to the original equation.

We therefore must check solutions to eliminate extraneous solutions. We will revisit this idea in more detail in Section 2.8.

$\sqrt{x}+x$ evaluated at $x=\frac{3+\sqrt{5}}{2}$ is $4.23\neq 1$, so this is an extraneous solution.

$\sqrt{x}+x$ evaluated at $x=\frac{3-\sqrt{5}}{2}$ is $1=1$, so this is a solution.