"Present Value"

What would you rather have?

   a) a dollar today
   b) a dollar ten years from now

Why it matters, and how it fits in with the course ...

In macro, GDP fluctuations are the focus

Of the components of GDP, Investment is the most volatile

   When we’re talking Investment, the key thing is:
   Costs & Benefits not synchronized

   Costs: upfront
   Benefits: delayed, protracted

So how do you compare today’s $s with year 2020 $s?

The answer is going to help us understand lots of things,
   among them Investment
Start with this question:

"What is the most I'd be willing to pay today, in exchange for $1 a year from today?"

*Note: You can "manufacture" that $1 (to be delivered after 1 year) yourself, by "buying" a savings deposit at any bank downtown.*

How much would it cost you to "manufacture" $1 post-dated 1 year?

If you invest \(X\) today at interest rate \(i\), then after one year you will have

\[X(1+i)\]

So how much would you need to invest today in order to accumulate $1 after 1 year? Solve for \(X\)

\[X(1+i) = 1\]

i.e.

\[X = 1/(1+i)\]

Plugging in a realistic interest rate (say, \(i = .05\))

\[X = 1/(1+.05) \approx .95\]

If the interest rate were .05 (5%), you could "manufacture" a 1-year post-dated dollar for just 95 cents today

"The present value of a dollar delivered one year in the future is 95 cents." (You’d never pay more than $.95 today for $1 after 1 year, given interest of 5%)
How much would it cost you today to produce a dollar two years from now?

Investing $X$, @ interest rate $i$, after 1 year we have

$$X(1+i)$$

Re-investing our $X(1+i)$ after 1 year (again at interest rate $i$) leaves us after 2 years with

$$[X(1+i)](1+i)$$

or

$$X(1+i)^2$$

To find out how much we'd need to invest today at interest rate $i$ to produce $1$ after 2 years, solve the following for $X$

$$X(1+i)^2 = 1$$

i.e.

$$X = 1/(1+i)^2$$

Using the same realistic interest rate (5%)

$$X = 1/(1.05)^2 \approx .91$$

If you invested $.91 today at interest rate 5%, you'd have $1 after two years.

⇒ The present value of a dollar post-dated two years is $.91
What's the present value (PV) of $k$ dollars post-dated $n$ years?

$$PV = \frac{k}{(1+i)^n}$$

Define:

Net Present Value $\equiv$ (Present value of benefits) - (Present Value of Costs)

Decision rule:

If the NPV (net present value) of an investment project is positive, the investment is worthwhile. If NPV is negative, the project is not worth investing in.
**Problem:** You stand to inherit $1 million when you turn 40 (i.e. in 20 years). You want it now. What’s it worth in today’s $s?

Answer:

\[ PV = \frac{1}{(1 + i)^{20}} \]

let i = .05  \Rightarrow PV = 376,889

let i = .1  \Rightarrow PV = 148,000

let i = .15  \Rightarrow PV = 61,000

**Important Fact:**

**Present Value of future claims is negatively related to interest rates**
Problem: The Company offers the union either a) a one-time up-front $1300 per-employee bonus, or b) a permanent $250/year raise and a one-time $500 retirement bonus. Assume the average worker has 5 years left until retirement. Which option should the average worker favor if

i) \( i = 0.05 \)

\[
PV = \frac{250}{1.05^1} + \frac{250}{1.05^2} + \frac{250}{1.05^3} + \frac{250}{1.05^4} + \frac{250}{1.05^5} + \frac{500}{1.05^5}
\]

\[
= 238.09 + 226.76 + 215.95 + 205.67 + 195.88 + 391.76
\]

\[
= 1474.11
\]

Best option: take the permanent increase

ii) \( i = 0.1 \)

\[
PV = \frac{250}{1.1^1} + \frac{250}{1.1^2} + \frac{250}{1.1^3} + \frac{250}{1.1^4} + \frac{250}{1.1^5} + \frac{500}{1.1^5}
\]

\[
= 227.27 + 206.61 + 187.82 + 170.75 + 155.23 + 310.46
\]

\[
= 1258.14
\]

Best option: take the one-time payout
**Problem:** The lottery claims it pays out as much as it takes in, and in a literal sense, this is true. However, what it pays out, it pays out over 20 years. Suppose the Lottery takes in $10 million in Lotto ticket revenue, and the winner gets paid $500,000 a year for 20 years, starting 1 year after the winning date.

i) Write an expression for the PV of the lottery winner's revenue stream.

ii) What profit does the Lottery make on this transaction? Explain.

Note: other ways the state profits off the lottery include

a) The sum they pay out is less than the sum they take in
b) Lottery winnings are taxable