Chapter 12

Note: wherever possible, values used in the solutions below are taken directly from the SAS output provided in the text.

1. a For smokers: $\hat{Y} = 79.225 + 20.118X$

For non-smokers: $\hat{Y} = 49.312 + 26.303X$

b $H_0: \beta_{1SMK} = \beta_{1\overline{SMK}}$ $\quad H_A: \beta_{1SMK} < \beta_{1\overline{SMK}}$

We need the following values from the output provided:

	n	$\hat{\beta}_0$	$\hat{\beta}_1$	$\overline{X}$	$\overline{Y}$	S^2_X	$S^2_{Y\|X}$
Non-smokers	15	49.312	26.303	3.478	140.8	0.176	48.274
Smokers	17	79.255	20.118	3.408	147.824	0.322	107.620

We also need $S^2_{p,Y|X}$, the pooled estimate of σ^2, which is determined as follows:

$$S^2_{p,Y|X} = \frac{(n_{\overline{S}} - 2)S^2_{Y|X_{\overline{S}}} + (n_S - 2)S^2_{Y|X_S}}{n_{\overline{S}} + n_S - 4} = \frac{(13)(48.274) + (15)(107.62)}{28} = 80.067$$

We also need

$$S^2_{(\hat{\beta}_{1\overline{SMK}} - \hat{\beta}_{1SMK})} = S^2_{p,Y|X}\left[\frac{1}{(n_{\overline{S}} - 1)S^2_{X_{\overline{S}}}} + \frac{1}{(n_S - 1)S^2_{X_S}}\right] = 80.067\left[\frac{1}{14(0.176)} + \frac{1}{16(0.322)}\right] = 48.04$$

The test statistic is then: $T = \dfrac{\hat{\beta}_{1\overline{SMK}} - \hat{\beta}_{1SMK}}{S_{(\hat{\beta}_{1\overline{SMK}} - \hat{\beta}_{1SMK})}} = \dfrac{26.303 - 20.118}{\sqrt{48.04}} = 0.892$

The test statistic follows a t distribution with 28 degrees of freedom (df) under H_0. The P-value is: $0.15 < P < 0.25$. At $\alpha = 0.05$ we do not reject H_0; the slopes for smokers and non-smokers are the same.

c $H_0: \beta_{0SMK} = \beta_{0\overline{SMK}}$ $\quad H_A: \beta_{0SMK} \neq \beta_{0\overline{SMK}}$

$$T = \frac{\hat{\beta}_{0\overline{SMK}} - \hat{\beta}_{0SMK}}{S_{(\hat{\beta}_{0\overline{S}} - \hat{\beta}_{0S})}} = \frac{49.312 - 79.255}{\sqrt{583.528}} = -1.24, \quad (28 \text{ df})$$

where $S^2_{(\hat{\beta}_{0\overline{SMK}} - \hat{\beta}_{0SMK})} = S^2_{p,Y|X}\left[\dfrac{1}{n_{\overline{S}}} + \dfrac{1}{n_S} + \dfrac{\overline{X^2_{\overline{S}}}}{(n_{\overline{S}} - 1)S^2_{X_{\overline{S}}}} + \dfrac{\overline{X^2_S}}{(n_S - 1)S^2_{X_S}}\right]$

$$= 80.067\left[\frac{1}{15} + \frac{1}{17} + \frac{3.478^2}{(14)(0.176)} + \frac{3.408^2}{(16)(0.322)}\right] = 583.528$$

P-value: $0.20 < P < 0.30$

At $\alpha = 0.05$ we do not reject H_0 and conclude that the two intercepts are equal.

d From the above tests, we would conclude that the straight lines for smokers and non-smokers are coincident since both tests failed to reject H_0.