Standard Error

When the standard deviation of a statistic is estimated from the data, the result is called the **standard error** of the statistic. The standard error of the sample mean is

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

How do you find s? First, we need find a sample variance s^2.

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right] \quad \text{Thus, } s = \sqrt{s^2}$$

The t Distributions

Suppose that an SRS of size n is drawn from an N(μ, σ) population. Then the **one-sample t statistic**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

has the **t distribution** with n-1 **degrees of freedom.**

The One-Sample t Confidence Interval

Suppose that an SRS of size n is drawn from a population having unknown mean μ A level C confidence interval for μ is

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where t* is the value for the t(n-1) density curve with area C between –t* and t*. This interval is exact when the population distribution is normal and is approximately correct for large n in other cases.

So the **Margin of error** for the population mean when we use the data to estimate σ is

$$t^* \frac{s}{\sqrt{n}}$$

Matched Pairs t procedures

One application of these one-sample t procedures is to the analysis of data from matched pairs studies. We compute the differences between the two values of a matched pair (often before and after measurements on the same unit) to produce a single sample value. The sample mean and standard deviation of these differences are computed.

Example 1. You want to rent an unfurnished one-bedroom apartment for next semester. You take a random sample of 10 apartments advertised in the local newspaper and record the rental rates. Here are the rents (in dollars per month):

505, 655, 600, 505, 450, 550, 520, 500, 650, 400

Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

$\bar{X} = 533.5, \; S = 82.16$

$df = 10 - 1 = 9$

$t^* = 2.262$

$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 533.5 \pm 2.262 \cdot \frac{82.16}{\sqrt{10}}$

$= 533.5 \pm 58.77$

$= (533.5 - 58.77, \; 533.5 + 58.77)$

$= (474.73, \; 592.27)$

Example 2. A random sample of 10 one-bedroom apartments from your local newspaper has these monthly rents (dollars):

505, 655, 600, 505, 450, 550, 520, 500, 650, 400

Do these data give good reason to believe that the mean rent of all advertised apartments is greater than \$500 per month? State Hypothesis, find the t statistic and its P-value, and state your conclusion.

$H_0: \mu = 500$ vs. $H_a: \mu > 500$

$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{533.5 - 500}{\frac{82.16}{\sqrt{10}}}$

$= \frac{33.5}{25.981} = 1.29$ ($df = 9$ From Table D)

$0.10 < p\text{-value} < 0.15$

Since p-value $> \alpha = 0.05$,

We do not reject H_0 at the 5% level.

Example 3. The one-sample t statistic from a sample of n=50 observations for the two-sided test of

$$H_0: \mu = 48$$
$$H_a: \mu \neq 48$$

two-sided Test

has the value t=1.65.

(a) What are the degrees of freedom for t?

$df = n-1 = 50-1 = 49$

(b) Locate the two critical values t* from Table D that bracket t. What are the right-tail probabilities p for these two values?

Choose closest df = 50

$1.299 < t^* < 1.676$

$0.05 <$ right tail prob < 0.10

(c) How would you report the P-value for this test?

$0.10 <$ P-value < 0.20

(d) Is the value t=1.65 statistically significant at the 5% level?

Since P-value $> \alpha = 0.05$,

we do not reject H_0 at the 5% level.

No, there is not statistically significant at the 5% level.

diff = Haiti − Factory

Example 4. Here are the data:

Factory	Haiti	Diff	Factory	Haiti	Diff
44	40	−4	45	38	−7
50	37	−13	32	40	8
48	39	−9	47	35	−12
44	35	−9	40	38	−2
42	35	−7	38	34	−4
47	41	−6	41	35	−6
49	37	−12	43	37	−6
50	37	−13	40	34	−6
39	34	−5	37	40	3

(a) Set up hypotheses to examine the question of interest to these researchers.

$H_0: \mu = 0$ vs $H_a: \mu < 0$

(b) Perform the significance test and summarize your results.

$\bar{x} = -6.1$, $s = 5.38$, $n = 18$ $\Rightarrow t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{-6.1 - 0}{5.38/\sqrt{18}} = -4.8$

$df = 17$. P-value $< 0.005 \Rightarrow$ P-value $< \alpha = 0.05$

Reject H_0 at the 5% level

(c) Find 95% confidence intervals for the mean at the factory, the mean five months later in Haiti, and for the change.

95% C.I for μ is

$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = -6.1 \pm 2.110 \cdot \frac{5.18}{\sqrt{18}}$

$\Rightarrow -6.1 \pm 2.68$

$\Rightarrow (-8.78, -3.42)$

Example 5. The table below gives the pretest and posttest scores on the MLA listening test in Spanish for 10 high school Spanish teachers who attended an intensive summer course in Spanish.

Subject	Pretest	Posttest	Diff
1	30	29	−1
2	28	30	2
3	31	32	1
4	26	30	4
5	20	16	−4
6	30	25	−5
7	34	31	−3
8	15	18	3
9	28	33	5
10	20	25	5

$\bar{x} = 0.7$, $s = 3.743$, $n = 10$, $df = 9$

(a) We hope to show that attending the institute improves listening skills. State an appropriate H_0 and H_a. Be sure to identify the parameters appearing in the hypotheses.

$H_0: \mu = 0$

$H_a: \mu > 0$

μ = mean diff of posttest − pretest.

(b) Carry out a test. Can you reject H_0 at the 5% significance level?

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.7 - 0}{3.743/\sqrt{10}} = 0.59.$

P-value > 0.25

Since P-value $> \alpha = 0.05$, we can't reject H_0 at the 5% level.

(c) Give a 90% confidence interval for the mean increase in listening score due to attending the summer institute.

90% C.I for μ is

$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

$= 0.7 \pm 1.833 \cdot \frac{3.743}{\sqrt{10}}$

$= 0.7 \pm 2.17$

$= (-2.1, 2.24)$ ✓