CSci 4651 Spring 2012
Problem Set 3: λ-calculus

Due Friday, February 17th in class.

Problem 1 (6 points).
Question 1. Which of the following terms are α-equivalent to $\lambda x.\lambda y.xyz$?

1. $\lambda y.\lambda x.yxz$
2. $\lambda x.\lambda x.xxz$
3. $\lambda z.\lambda y.zxz$
4. $\lambda y.\lambda z.yzx$

Please explain your answer.

Question 2. For each term in Question 1 (including the original term in the question itself) please write the set of its free variables.

Problem 2 (10 points). Question 1. For each of the terms show their evaluation in the **call by value** calculus. Continue the evaluation until you either reach the normal form, or, if the term doesn't have a normal form, until you can demonstrate that the term diverges.

1. $(\lambda x.xx)(\lambda y.2 + 3)$
2. $(\lambda x.x + x)((\lambda y.y)(2 + 3))$
3. $(\lambda x.\lambda y.x)3$
4. $(\lambda x.xxx)(\lambda x.xx)$
5. $(\lambda z.y)((\lambda x.xxx)(\lambda x.xxx))$

Question 2. For any of the above terms if there is a different evaluation in the call by name calculus, show that evaluation.

Question 3. For all terms above that go into an infinite reduction in the call by value calculus, is there an evaluation in the call by name calculus that stops? If yes, show it.

Problem 3 (2 points). Consider the call-by-name λ-calculus. Given a program

$$(\lambda x.\lambda y.((\lambda z.y)5)x)M,$$

where M is any term, can you replace this program by $(\lambda x.\lambda y.yx)M$ without changing the program's behavior? Why or why not?