IMPORTANT, please read before you start working on the problems: This problem set uses Java-like syntax for convenience. However, this is NOT Java code. Read each problem carefully to understand which model you are working in (pass-by-value vs. pass-by-reference, static vs. dynamic scope rules, etc.). Assume that all code given below is syntactically correct and does not cause any compilation errors. Assume that `print("x = " + x)` is a printing command which works like `System.out.println("x = " + x)` in Java for all types of variables used in this problem set.

Problem 1 (5 points). Consider the following code (the lines are numbered for easy reference):

1. int x = 3;
2. {
3.     int y = 5 + x;
4.     {
5.         int x = 2 + y;
6.     }
7.     x = y;
8. }

Question 1. Draw the program stack right after line 5 gets executed.

Question 2. What is the final value of the global `x` (the one defined on line 1) in this program fragment?

Problem 2 (5 points). Consider the call `f(2, -1)` to the following function:

```java
int f (int x, int y) {
    int z = 0;
    int w = x * x;
    if (y < 0) {
        int z = -w;
        print("z = " + z);
    }
    return z;
}
```

Question 1. Draw the program stack at the point right after the line `int z = -w;` is executed.
**Question 2.** What will be printed by the `print` statement? What will be the value returned by the function? Explain your answers using the stack diagram from Question 1.

**Problem 3 (10 points).** Which of the following Java methods are tail-recursive? If a method is not tail-recursive, please write its tail-recursive version (make sure to show the first call to the method that initializes the extra parameters, if any). Feel free to test your code; you may submit electronic solutions. The source code is also linked from the assignments page for copy/pasting.

```java
public class TailRecursion {
    public static void main(String[] args) {
        int[] A = {3, 6, 7, 5, 4};
        int[] B = {3, 6, 5, 7, 9};
        System.out.println("sum_array(A,0) = " + sum_array(A,0));
        System.out.println("find(A,0,5) = " + find(A,0,5));
        System.out.println("find(A,0,2) = " + find(A,0,2));
        System.out.println("equal(A,B,0) = " + equal(A,B,0));
        System.out.println("equal(A,A,0) = " + equal(A,A,0));
        System.out.println("equal(A, new int[2],0) = " + equal(A, new int[2],0));
        System.out.println("to_string_reverse(A,0)" + to_string_reverse(A,0));
    }

    public static int sum_array(int[] A, int i) {
        if (i < A.length) return (A[i] + sum_array(A, i+1));
        return 0;
    }

    public static boolean find(int[] A, int i, int x) {
        if (i >= A.length) return false;
        if (A[i] == x) return true;
        return find(A, i+1, x);
    }

    public static boolean equal(int[] A, int[] B, int i) {
        if (i == A.length) {
            if (i == B.length) return true;
            return false;
        }
        return (A[i] == B[i]) ? equal(A, B, i+1) : false;
    }

    public static String to_string_reverse(int[] A, int i) {
        if (i < A.length) {
            return to_string_reverse(A, i+1) + " " + A[i];
        }
    }
}
```
return "";
}
}

Question 2. Describe how a tail-recursive function may be optimized. Explain why the optimized program is more efficient (what overhead has been eliminated by the optimization?).

Problem 4 (8 points). Consider the following function. Notice that the first parameter is passed by reference, and the second one is passed by value.

```c
int f(int &n, int m) {
    n = n + 1;
    m = m + 1;
    return n + m;
}
```

Question 1. Which of the following are valid calls to this functions? Assume that all variables used below are of type `int` and `A` is an array of integers, and all elements of the array referenced below are within the array boundary. Explain your answers briefly.

- `f(x + 1, 5)`
- `f(A[i+1], A[i])`
- `f(2, 3)`
- `f(x, f(x, y))`

Question 2. For the following function call please draw the function stack right after `f` was called and right before `f` returns. What will be printed by the program? Justify your answer using the stack diagram.

```c
... main (...) {
    int x = 2;
    int y = 0;
    y = f(x, x);
    print x;
    print y;
}
```

Problem 5 (10 points). Consider the following program, where `main` is the first function in program execution. Parameters are passed by value.

```c
int x = 2;
int y = 3;

void f(int n) {
    x = x + n;
    y = y - n;
```
```c
void main () {
    int x = 1;
    f(1);
    {
        int y = 5;
        f(x);
    }
}

Question 1. Assuming dynamic scope rules, draw the program stack at two points in execution: right before the call f(1) returns and right before the call f(x) returns. Show values of all variables in the stack pictures.

Note that for the second function call the stack will have both kinds of blocks: in-line blocks and those associated with a function.

Question 2. What are the final values of global x and y in the case of dynamic scope rules? Use the stack pictures to explain your answer.

Question 3 and 4. The same as 1 and 2, but for static (lexical) scope rules.

Problem 6 (8 points). Consider the following program where main is the first function in program execution. Assume the static scope rules. int -> int is the type of functions from an integer to another integer.

int x = 0;

int f(int a) {
    if (a == 1) return x;
    else {
        x = x + a;
        return f(a - 1);
    }
}

void g (int -> int h) {
    int x = 5;
    print(h(2));
}

void main () {
    g(f);
}

Question 1. Note that f is recursive. Draw the program stack and function closures when all activation records for f are pushed on the stack.

Question 2. What is going to be printed in the program? Use the stack picture
to explain your answer.

**Problem 7 (8 points).** Assume the static scope rules and notations as in the previous problem. Consider the following code fragment, where \( f \) returns a function \( g \). The function type \( \text{void} \rightarrow \text{void} \) means that the function takes no parameters and does not return a value.

```c
1. void -> void f () {
2.     int x = 0;
3.     return ( void g() { x = x + 1; } );
4. }
5. void main () {
6.     void -> void h = f();
7.     h();
8.     void -> void j = f();
9.     j();
10. }
```

Draw the program stack and all the function closures at the following points of the program execution:

1. Right before line 7 is executed.
2. Right after line 7 is executed.
3. Right after line 8 is executed.
4. Right after line 9 is executed.

Show the values of all variables.