CSci 4651 Fall 2003
Problem Set 3: λ-calculus and denotational semantics
Due Friday, October 3rd

Problem 1. Consider the following grammar for boolean expressions:

$$\begin{array}{lcl} e & \rightarrow & b \mid v \mid e \wedge e \mid e \vee e \mid \neg e \\ b & \rightarrow & \text{true} \mid \text{false} \\ v & \rightarrow & x \mid y \mid z \end{array}$$

Without making any assumptions about the precedence of the operations, please answer the following about each of the expressions below:

- Is the expression a part of the language defined by the grammar?
- If yes, please draw all possible parse trees for it in the grammar.

The expressions are as follows:

1. $x \wedge \text{false}$
2. $x \neg y \wedge z$
3. $x \vee x \wedge y$
4. $x \vee \neg \text{true}$

Is the grammar ambiguous? Please justify your answer.

Problem 2. Which of the following terms are α-equivalent (i.e. equivalent up to renaming of the bound variables) to $\lambda x.\lambda y.xyz$?

1. $\lambda y.\lambda x.yxz$
2. $\lambda x.\lambda x.xxz$
3. $\lambda z.\lambda y.zxz$
4. $\lambda y.\lambda z.yzx$

Please explain your answer.

Problem 3. For each of the terms below show its step-by-step evaluation in the call-by-value or the call-by-name λ-calculus. Continue the evaluation until you either reach the normal form, or, if the term doesn't have a normal form, until you can demonstrate that the term diverges. If the evaluation is the same in the two calculi, then you only need to show one of them.

1. $(\lambda x.xx)(\lambda y.2+3)$
2. $(\lambda x.\lambda y.x)3$
3. $(\lambda x.xxx)(\lambda x.xxx)$

4. $(\lambda z.y)((\lambda x.xxx)(\lambda x.xxx))$.

Problem 4. Consider the call-by-name λ-calculus. Given a program

$$(\lambda x.\lambda y.((\lambda z.y)5)x)M,$$

where M is any term, can you replace it by $(\lambda x.\lambda y.yx)M$ without changing the program's behavior? Why or why not?

Problem 5. Exercise 4.8 on p. 85.

Problem 6. Exercise 4.9 on p. 85, Part (a) only.