CSci 4554 Assignment 7

Due Friday, March 28th in class

Problem 1 (16 points). Please characterize the following sets with the given operation as:

- a group (if it is a group, what is the identity?),
- abelian (=commutative) group,
- a finite group.

Please explain your answers. A set may belong to more than one category or to none at all.

The sets to consider are:

1. $(\mathbb{Z}, -)$
2. $(\mathbb{Z}, *)$
3. $(\mathbb{R}, *)$
4. A set of strings over a given alphabet with a string concatenation operation.
5. A set $\{0, 1\}$ with $\oplus$ ("exclusive or") operation.
6. A set $\{0, 1\}$ with $\vee$ ("or") operation.
7. A set of strings of 0 and 1 with bitwise $\oplus$ ("exclusive or") operation. Strings are of an arbitrary length, a shorter string is assumed to be padded with zeros on the right before $\oplus$ is applied.
8. A set of strings of 0 and 1 with bitwise $\vee$ ("or") operation.

Problem 2 (20 points). Please characterize the following sets with the two given operation as:

- a ring,
- a field

Please explain your answers. A set may belong to more than one category or to none at all.

The sets to consider are as follows. The first operation plays the role of "addition" and the second one of "multiplication".

1. $(\mathbb{Z}, +, *)$
2. $(\mathbb{R}, +, *)$

3. $(\mathbb{Z}_{25}, +, *)$
4. A set $\{0, 1\}$ with $\vee$ and $\wedge$.
5. A set $\{0, 1\}$ with $\wedge$ and $\vee$.

Problem 3 (15 points). Polynomials over algebraic structures. In each question you are given an algebraic structure and a property of two polynomials. Please give examples of such polynomials if they exist and demonstrate by a computation that the property holds. If they do not exist, please explain why.

1. In $\mathbb{Z}_4[x]$, are there polynomials $f(x)$, $g(x)$ s.t. $\deg(f+g) < \max\{\deg(f), \deg(g)\}$?
2. In $\mathbb{Z}_5[x]$, are there polynomials $f(x)$, $g(x)$ s.t. $\deg(f+g) < \max\{\deg(f), \deg(g)\}$?
3. In $\mathbb{Z}_4[x]$, are there polynomials $f(x)$, $g(x)$ s.t. $\deg(fg) < \deg(f) + \deg(g)$?
4. In $\mathbb{Z}_5[x]$, are there polynomials $f(x)$, $g(x)$ s.t. $\deg(fg) < \deg(f) + \deg(g)$?
5. In $\mathbb{Z}_5[x]$, are there polynomials $q(x), r(x)$ with $\deg(r) < 2$ that solve the equation $3x^3 + x^2 + 2x + 1 = (2x^2 + 3)q + r$?

Problem 4 (9 points). Exercise 7.9 p. 243.