CSci 3501 Assignment 3

Due Friday, September 23rd in class

Problem 1 (5 points) Consider a search algorithm for a sorted array that works as follows: given an array of n elements and a search value v, it takes an element at the index $\frac{n}{3}$ and an element at the index $\frac{2n}{3}$ (with appropriate rounding when needed). It compares v to the two elements and, if no match is found, makes a recursive call on one third of the array. For instance, if v is less than the smaller "pivot" value then the call is on the first third; if v is larger than the smaller pivot but smaller than the other one then the recursive call is on the middle third of the array, and so on.

Write the recurrence for the algorithm. What is the solution of the recurrence and why? You don't need a formal proof, just an explanation.

Would you consider this algorithm to be more or less efficient than binary search? Please explain your answer.

Problem 2 (15 points). Use the substituion method (i.e. a proof by induction) to prove the following:

1. The solution of the recurrence $T(n) = T(\frac{n}{2}) + n$ is $O(n)$.
2. The solution of the recurrence $T(n) = 2T(\frac{n}{2}) + n^2$ is $O(n^2)$.
3. The solution of the recurrence $T(n) = 3T(\frac{n}{3}) + 1$ is $O(n)$.

Assume the base case $T(1) = 1$ for all recurrences. You may ignore the precise handling of base cases.

Problem 3 (8 points). Use the recurrence tree method to solve the following recurrences:

1. $T(n) = T(n-2) + n$, the base cases are $T(1) = T(0) = c$ (why do we need two base cases here?)
2. $T(n) = T(\frac{n}{3}) + 1$, the base case $T(1) = c$.

Problem 4 (10 points). Exercise 4-1 p. 107, parts a, b, c, d, e. Use the Master Theorem, show all your computations. If the Master Theorem is not applicable, please clearly explain why (you don't need to solve the problem in this case).

In your final result please do not compute the values of logarithms, i.e. leave expressions like $\log_2 7$ as is. The textbook section on logarithms on p. 56 gives helpful logarithms identities.

Problem 5 (4 points). Exercise 4-3 p. 108 part f. Hint: use a similar recurrence that **can** be solved by the Master Theorem to **estimate** the answer and then **prove** your answer using the substitution method. A bound that is correct, but not asymptotically tight, will get a partial credit.