CSci 1302 Assignment 6

Due Wednesday, February 25th in class

Problem 1 (16 points: 1,2 are 2 points each, the rest are 3 points each). Assume the following:

1. A chess team A consists of Adam, Alice, and Ann. A(x) means that the person x is on the team A.
2. A chess team B consists of Bob and Beth. B(x) means that the person x is on the team B.
3. The universal set for the problem is the set of all five chess players.
4. The predicated $\text{def}(x, y)$ means that x has defeated y at least once. Some people never played against each other, so no comparison is given for such pairs. The following are true statements:
   (a) Adam has defeated Ann and Alice.
   (b) Alice has defeated Ann.
   (c) Beth has defeated Adam and Bob.
   (d) Bob has defeated Alice.
   (e) Ann has defeated Alice.

   Note that since no person defeated themselves, $\text{def}(x, x)$ is false for any x in the domain.

Based on the information above, are the following true or false statements? Prove your answers by showing all instances (single elements or pairs) necessary to prove or disprove the statement. When considering all possible pairs, it may be convenient to organize your answers as a table. You may show only a part of the table if it is sufficient to prove or disprove the statement.

1. $\exists x.A(x) \rightarrow \text{def}(x, Beth)$
2. $\forall x.\text{def}(Adam, x) \vee \text{def}(Beth, x)$
3. $\forall x.\exists y.A(x) \rightarrow \text{def}(y, x)$
4. $\exists x.\forall y.\text{def}(x, y) \vee \text{def}(y, x)$
5. $\exists x.\exists y.\text{def}(x, y) \wedge \text{def}(Adam, x)$
6. $\forall x.\forall y.A(x) \vee B(y)$

Problem 1A (Extra credit, 3 points) Use the system in Problem 1 to give an example of a predicate $p(x, y)$ such that $\forall x.\exists y.p(x, y)$ is true, but $\exists y.\forall x.p(x, y)$ is not. You may combine the given predicates in any way you want, but do not add any new predicates.

Problem 2 (6 points). Exercise 40c, d, f, g, h, i p. 110.

Problem 3 (6 points). Exercises 54, 55, 56 p. 110.

Problem 4 (12 points). Prove the following arguments. The domain for all problems is $\mathbb{Z}$ - the set of all integers.

$$
\begin{array}{lll}
A. & 1. & \forall x. x \cdot 1 = x \\
\hline
 & & \therefore \forall x. \exists y. x \cdot y = x
\end{array}
$$

$$
\begin{array}{lll}
B. & 1. & \forall x. (x \neq 1 \wedge x \neq 0) \rightarrow x^2 > x \\
 & 2. & \exists y. y \neq 1 \wedge y \neq 0 \\
\hline
 & & \therefore \exists z. z^2 > z
\end{array}
$$

Hint for problem C: when introducing the existential quantifier, replace one occurrence of the constant by a variable, but not the other:

$$
\begin{array}{lll}
C. & 1. & \forall z. isDivisible(z, 1) \wedge isDivisible(z, z) \\
\hline
 & & \therefore \exists y. isDivisible(y, 33)
\end{array}
$$