CSci 1302 Assignment 5
Due Wednesday, February 18th in class

Problem 1 (6 points). Exercise 34 pp. 57.

Problem 2 (4 points). Exercises 3, 9, and 18 p. 73.

Problem 3 (4 points). Exercises 24, 26, 28, 30 p. 73.

Problem 4 (6 points). Exercises 34, 35, 36 p. 73.

Problem 5 (3 points). Exercises 39, 42, 45 p. 73.

For the following two problems assume the domain (i.e. the universal set) to be the set $\mathbb{Z}$ of all integers: positive, negative, and zero. We use binary predicates $x < y$, $x \leq y$ and the like, $isOdd(x)$, $isEven(x)$, and $isDivisibleBy(x, y)$, the latter meaning that x is divisible by y.

Problem 6 (10 points). Translate the following formuals to English, indicate whether each one is true or false, and briefly justify your answer.

1. $\tilde{\ }\forall x.(x^2 > 0) \vee (x = 0)$

2. $\exists x.x^2 \leq x$

3. $\forall x.isOdd(x) \rightarrow isEven(x)$

4. $\exists x.(x \leq 2) \rightarrow (isOdd(x) \wedge isEven(x))$

5. $\exists x.(x \leq 2) \leftrightarrow isOdd(x)$

6. $\exists x.\exists y.(x > 2) \wedge (x + y < 0)$

7. $\exists x.\tilde{\ }isDivisible(x, x)$

8. $\forall x.\exists y.isDivisible(x, y)$

9. $\exists x.\forall y.isDivisible(y, x)$

10. $\forall x.\exists y.isOdd(x) \rightarrow isOdd(y)$

Problem 7 (10 points). Write the following sentences as quantified formulas. Note that some of these formulas need more than one quantifier.

1. Every number is divisible by 1.

2. Some numbers are divisible by 3.

3. Not all numbers are divisible by 3.

4. No odd number is divisible by 2.

5. No number is greater than itself.

6. Squares of odd numbers are odd.

7. No squares of even numbers are prime.

8. Every number is divisible by some number.

9. Some numbers are squares of some numbers (don't use the predicate $isSquare(x)$).

10. No matter what pair of numbers you take, you can find a number that they both are divisible by.