# CSci 1302 Assignment 6

**Due Wedn., March 9th in class**

**Problem 1 (28 points).** Prove the following arguments. The domain for all problems is $\mathbb{Z}$ - the set of all integers.

*A.* 1. $\forall x.x \cdot 1 = x$

$\therefore \forall x.\exists y.x \cdot y = x$

*B.* 1. $\forall x.(x \neq 1 \wedge x \neq 0) \rightarrow x^2 > x$
2. $\exists y.y \neq 1 \wedge y \neq 0$

$\therefore \exists z.z^2 > z$

**Hint for problem C:** when introducing the existential quantifier, replace one occurrence of the constant by a variable, but not the other:

*C.* 1. $\forall z.isDivisible(z, 1) \wedge isDivisible(z, z)$

$\therefore \exists y.isDivisible(y, 33)$

*D.* 1. $\forall x.\forall y.(x > y) \vee (y > x) \vee (x = y)$
2. $\tilde{}(5 > 5)$

$\therefore 5 = 5$

*E.* 1. $\forall x.\forall y.\exists z.x + y = z$

$\therefore \forall x.\exists z.x + x = z$

*F.* 1. $\forall x.isPrime(x) \leftrightarrow (\forall y.isDivisible(x, y) \rightarrow (y = 1 \vee y = x))$
2. $isDivisible(9, 3)$
3. $3 \neq 1 \wedge 3 \neq 9$

$\therefore \tilde{}isPrime(9)$

*G.* 1. $\forall x.odd(x) \leftrightarrow (\tilde{}\exists y.x = 2 \cdot y)$
$4 = 2 \cdot 2$

$\therefore \tilde{}odd(4)$