

CSci 1302 Assignment 6

Due Fri., March 5th in class

Problem 1 (6 points). Exercise 40c, d, f, g, h, i p. 110.

Problem 2 (6 points). Exercises 54, 55, 56 p. 110.

Problem 3 (28 points). Prove the following arguments. The domain for all problems is \mathbb{Z} - the set of all integers.

$$\begin{array}{l} A. \quad 1. \quad \forall x.x \cdot 1 = x \\ \hline \quad \quad \therefore \forall x.\exists y.x \cdot y = x \end{array}$$

$$\begin{array}{l} B. \quad 1. \quad \forall x.(x \neq 1 \wedge x \neq 0) \rightarrow x^2 > x \\ \quad 2. \quad \exists y.y \neq 1 \wedge y \neq 0 \\ \hline \quad \quad \therefore \exists z.z^2 > z \end{array}$$

Hint for problem C: when introducing the existential quantifier, replace one occurrence of the constant by a variable, but not the other:

$$\begin{array}{l} C. \quad 1. \quad \forall z.isDivisible(z, 1) \wedge isDivisible(z, z) \\ \hline \quad \quad \therefore \exists y.isDivisible(y, 33) \end{array}$$

$$\begin{array}{l} D. \quad 1. \quad \forall x.\forall y.(x > y) \vee (y > x) \vee (x = y) \\ \quad 2. \quad \sim(5 > 5) \\ \hline \quad \quad \therefore 5 = 5 \end{array}$$

$$\begin{array}{l} E. \quad 1. \quad \forall x.\forall y.\exists z.x + y = z \\ \hline \quad \quad \therefore \forall x.\exists z.x + x = z \end{array}$$

$$\begin{array}{l} F. \quad 1. \quad \forall x.isPrime(x) \leftrightarrow (\forall y.isDivisible(x, y) \rightarrow (y = 1 \vee y = x)) \\ \quad 2. \quad isDivisible(9, 3) \\ \quad 3. \quad 3 \neq 1 \wedge 3 \neq 9 \\ \hline \quad \quad \therefore \sim isPrime(9) \end{array}$$

$$\begin{array}{l} G. \quad 1. \quad \forall x.odd(x) \leftrightarrow (\sim \exists y.x = 2 \cdot y) \\ \quad \quad 4 = 2 \cdot 2 \\ \hline \quad \quad \therefore \sim odd(4) \end{array}$$