CSci 1302 Assignment 6
Due Fri., March 7th in class

Problem 1 (10 points). Consider a set $C$ of $n$ software development companies and a set $P$ of $n$ programmers looking for jobs. Each company ranks all of the programmers in order of preference for hiring (no two programmers get the same ranking), with 1 being the highest and $n$ being the lowest ranking. The ranking is given by the function $r$ so that, for instance, $r(c_1, p_1) = 3$ means that the company $c_1$ ranks the programmer $p_1$ as the third from the top.

Likewise, each programmer ranks the companies according to his/her preferences (no two companies get the same rankings). The ranking is given by the function $r'$. As an example, $r'(p_1, c_1) = 2$ means that the programmer $p_1$ ranks the company $c_1$ as his/her second choice.

Use quantified formulas to define the preference systems described below. Use letters $c, a, b$ to denote companies and letters $p, q, r$ to denote programmers.

As an example, consider: there is at least one programmer who is the first choice in his/her first choice company. Solution: $\exists p \exists c (\forall q.r(c, q) \geq r(c, p)) \land (\forall a.r'(p, a) \geq r'(p, c))$ (note that higher ranking numbers indicate lower preference!). You can also express this easier using the fact that the highest ranking is 1 as $\exists p \exists c r(c, p) = 1 \land r'(p, c) = 1$.

Express the following properties using quantifiers:

1. All companies rank all of the programmers in the same order.
2. No company gives the highest preference to a programmer who selected that company as their first choice.
3. No two programmers have the same first choice of a company.
4. If all programmers get their worst choice company, some companies will end up with someone who is their first choice.
5. There is a company that, given a choice between programmers $p$ and $q$, would prefer the one who ranks it lower than does the other programmer.

Problem 2 (6 points). Exercise 40c, d, f, g, h, i p. 110.

Problem 3 (6 points). Exercises 54, 55, 56 p. 110.

Problem 4 (28 points). Prove the following arguments. The domain for all problems is $\mathbb{Z}$ - the set of all integers.
A. 1. \( \forall x. x \cdot 1 = x \)
\[ \therefore \forall x. \exists y. x \cdot y = x \]

B. 1. \( \forall x. (x \neq 1 \land x \neq 0) \rightarrow x^2 > x \)
2. \( \exists y. y \neq 1 \land y \neq 0 \)
\[ \therefore \exists z. z^2 > z \]

Hint for problem C: when introducing the existential quantifier, replace only one occurrence of the constant by a variable, but not the other:

C. 1. \( \forall z. isDivisible(z, 1) \land isDivisible(z, z) \)
\[ \therefore \exists y. isDivisible(y, 33) \]

D. 1. \( \forall x. \forall y. (x > y) \lor (y > x) \lor (x = y) \)
2. \( \sim (5 > 5) \)
\[ \therefore 5 = 5 \]

E. 1. \( \forall x. \forall y. \exists z. x + y = z \)
\[ \therefore \forall x. \exists z. x + x = z \]

F. 1. \( \forall x. isPrime(x) \leftrightarrow (\forall y. isDivisible(x, y) \rightarrow (y = 1 \lor y = x)) \)
2. \( isDivisible(9, 3) \)
3. \( 3 \neq 1 \land 3 \neq 9 \)
\[ \therefore \sim isPrime(9) \]

G. 1. \( \forall x. odd(x) \leftrightarrow (\sim \exists y. x = 2 \cdot y) \)
4 = 2 \cdot 2
\[ \therefore \sim odd(4) \]