CSci 1302 Assignment 6

Due Fri., March 7th in class

Problem 1 (10 points). Consider a set C of n software development companies and a set P of n programmers looking for jobs. Each company ranks all of the programmers in order of preference for hiring (no two programmers get the same ranking), with 1 being the highest and n being the lowest ranking. The ranking is given by the function r so that, for instance, $r(c_1, p_1) = 3$ means that the company c_1 ranks the programmer p_1 as the third from the top.

Likewise, each programmer ranks the companies according to his/her preferences (no two companies get the same rankings). The ranking is given by the function r'. As an example, $r'(p_1, c_1) = 2$ means that the programmer p_1 ranks the company c_1 as his/her second choice.

Use quantified formulas to define the preference systems described below. Use letters c, a, b to denote companies and letters p, q, r to denote programmers.

As an example, consider: there is at least one programmer who is the first choice in his/her first choice company. Solution: $\exists p \exists c (\forall q. r(c, q) \geq r(c, p)) \wedge (\forall a. r'(p, a) \geq r'(p, c))$ (note that higher ranking numbers indicate lower preference!). You can also express this easier using the fact that the highest ranking is 1 as $\exists p \exists c r(c, p) = 1 \wedge r'(p, c) = 1$.

Express the following properties using quantifiers:

1. All companies rank all of the programmers in the same order.
2. No company gives the highest preference to a programmer who selected that company as their first choice.
3. No two programmers have the same first choice of a company.
4. If all programmers get their worst choice company, some companies will end up with someone who is their first choice.
5. There is a company that, given a choice between programmers p and q, would prefer the one who ranks it lower than does the other programmer.

Problem 2 (6 points). Exercise 40c, d, f, g, h, i p. 110.

Problem 3 (6 points). Exercises 54, 55, 56 p. 110.

Problem 4 (28 points). Prove the following arguments. The domain for all problems is $\mathbb{Z}$ - the set of all integers.

$A.$ 1. $\forall x. x \cdot 1 = x$

$\therefore \forall x. \exists y. x \cdot y = x$

$B.$ 1. $\forall x. (x \neq 1 \wedge x \neq 0) \rightarrow x^2 > x$

2. $\exists y. y \neq 1 \wedge y \neq 0$

$\therefore \exists z. z^2 > z$

Hint for problem C: when introducing the existential quantifier, replace only one occurrence of the constant by a variable, but not the other:

$C.$ 1. $\forall z. isDivisible(z, 1) \wedge isDivisible(z, z)$

$\therefore \exists y. isDivisible(y, 33)$

$D.$ 1. $\forall x. \forall y. (x > y) \vee (y > x) \vee (x = y)$

2. $\tilde{}(5 > 5)$

$\therefore 5 = 5$

$E.$ 1. $\forall x. \forall y. \exists z. x + y = z$

$\therefore \forall x. \exists z. x + x = z$

$F.$ 1. $\forall x. isPrime(x) \leftrightarrow (\forall y. isDivisible(x, y) \rightarrow (y = 1 \vee y = x))$

2. $isDivisible(9, 3)$

3. $3 \neq 1 \wedge 3 \neq 9$

$\therefore \tilde{}isPrime(9)$

$G.$ 1. $\forall x. odd(x) \leftrightarrow (\tilde{}\exists y. x = 2 \cdot y)$

$4 = 2 \cdot 2$

$\therefore \tilde{}odd(4)$