

**CSci 1302 Assignment 6**  
**Updated March 8 (typos in part F)**  
**Due Wednesday, March 15**

**Problem 1 (25 points).** Prove the following arguments. The domain for all problems is  $\mathbb{Z}$  - the set of all integers.

$$A. \quad 1. \quad \forall x. \exists y. x^2 = y$$


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$$\therefore \exists y. 5^2 = y$$

$$B. \quad 1. \quad \forall x. (x \neq 1 \wedge x \neq 0) \rightarrow x^2 > x$$

$$2. \quad \exists y. y \neq 1 \wedge y \neq 0$$


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$$\therefore \exists z. z^2 > z$$

**Hint:** when introducing the existential quantifier, replace only one occurrence of the constant by a variable, but not the other:

$$C. \quad 1. \quad \forall z. isDivisible(z, 1) \wedge isDivisible(z, z)$$


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$$\therefore \exists y. isDivisible(y, 33)$$

$$D. \quad 1. \quad \forall x. \forall y. x > y \vee y > x \vee x = y$$

$$2. \quad \sim(5 > 5)$$


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$$\therefore 5 = 5$$

$$E. \quad 1. \quad \forall x. \forall y. \exists z. x + y = z$$


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$$\therefore \forall x. \exists z. x + x = z$$

$$F. \quad 1. \quad \forall x. isPrime(x) \leftrightarrow (\forall y. isDivisible(x, y) \rightarrow (y = 1 \vee y = x))$$

$$2. \quad isDivisible(9, 3)$$

$$3. \quad 3 \neq 1 \wedge 3 \neq 9$$


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$$\therefore \sim isPrime(9)$$

$$G. \quad 1. \quad \forall x. \exists y. x > y$$

$$\forall x. \forall y. x > y \rightarrow x^2 > y^2$$


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$$\therefore \forall x. \exists y. x^2 > y^2$$

$$H. \quad 1. \quad \exists x. \forall y. isDivisible(y, x)$$


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$$\therefore \forall z. \exists w. isDivisible(z, w)$$