CSci 1302 Assignment 7
Due Wedn., March 17, 2004

This problem set uses some of the following predicates on integer numbers:

- Unary: $prime(x), even(x), odd(x)$ mean "x is prime", "x is even", "x is odd", respectively. $thelargest(x)$ means that x is the largest of all numbers.

- Binary: $equal(x, y)$ means "x is equal to y", $greater(x, y)$ means "$x > y$", $divisible(x, y)$ means "x is divisible by y".

- Ternary: $sum(x, y, z)$ means that $x = y + z$.

While the meaning of the predicates is not important for the proofs, it might be helpful in providing intuition about the statements.

Use subscripts (such as $x_\exists$ and $x_\forall$) to distinguish between unknown and genuine variables after instantiation and specify the rules as we did in class.

Problem 1 (16 points). Prove the following arguments in the predicate logic.

1. $\forall x.even(x) \Leftrightarrow divisible(x, 2)$
   $\exists x.even(x)$
   _____
   $\exists x.\exists y.divisible(x, y)$

2. $\forall x.\forall y.\exists z.sum(x, y, z)$
   _____
   $\forall x.\exists y.sum(x, x, y)$

3. $\forall x.\forall y.greater(x, y) \lor greater(y, x)$
   $\forall x.\neg greater(x, x)$
   _____
   false

4. $\forall x.thelargest(x) \Rightarrow \forall y.greater(x, y)$
   $\neg\exists w.\forall z.greater(w, z)$
   _____
   $\forall x.\neg thelargest(x)$

Problem 2 (8 points). Exercise 9.4 p. 131, parts 1, 2 only.

Problem 3 (8 points). Exercise 9.5 p. 132, parts 1 and 2 only.