CSci 1302 Assignment 5

Due Wedn., February 25th, 2004

This problem set uses the following predicates on integer numbers:

- Unary: $prime(x), even(x), odd(x)$ mean "x is prime", "x is even", "x is odd", respectively.
- Binary: $equal(x, y)$ means "x is equal to y", $greater(x, y)$ means "$x > y$", $divisible(x, y)$ means "x is divisible by y".
- Ternary: $sum(x, y, z)$ means that $x = y + z$.

Problem 1 (6 points). Recall exercise 5.6 (saints and liars). In each of the following statements, can you determine who the speaker is? Can you determine who Pat is? You must use truth tables to justify your result.

1. If I am a saint then Pat is a liar.
2. I am a liar, but if Pat is a saint, then I am a saint.
3. If Pat is a liar or I am a liar, then Pat is a saint.

Problem 2 (24 points). Write the following formulas in English. For each claim say whether it is true or false and briefly explain your answer.

1. $\forall x.\exists y.sum(x, y, y)$
2. $\forall x.\exists y.sum(y, x, x)$
3. $\exists x.\forall y.sum(x, x, y)$
4. $\forall x.even(x) \wedge \exists y.sum(x, y, y)$
5. $\forall x.odd(x) \Rightarrow \forall y.\neg sum(x, y, y)$
6. $\forall x.\forall y.\neg equal(x, y) \Rightarrow \forall z.divisible(x, z) \Rightarrow \neg divisible(y, z)$
7. $\forall x.\forall y.divisible(x, y) \Rightarrow \exists z.greater(z, x) \wedge divisible(z, y)$
8. $\forall x.\exists y.\forall z.greater(z, x) \Rightarrow divisible(z, y)$

Problem 3 (16 points). Write the following sentences as formulas in predicate logic.

1. Not all odd numbers are prime.
2. Not all prime numbers are odd.
3. All even numbers greater than 2 are divisible by 4.

4. No even number is greater than itself.
5. There is a number which is smaller than all even numbers.
6. There is the smallest even number.
7. Every even number can be represented as the sum of two odd numbers.
8. If a number is even then it is divisible by some number other than itself.