CSci 1302 Assignment 10

Due Fri., April 23rd, 2004

Notations: $\emptyset$ stands for the empty set, $\mathbb{N}$ is the set of natural numbers $(1, 2, 3, ...)$. $\mathbb{P}X$ stands for the power set of X.

Problem 1 (6 points). For each of the relations below:

- Write at least 5 elements that belong to the relation. If the relation has fewer than 5 elements, then write all of them.
- Find $dom(R)$.
- Find $ran(R)$.

The relations for the problem are as follows (all are considered on $\mathbb{N} \times \mathbb{N}$):

1. $R = \{(n, m) \mid n = m^2\}$
2. $R = \{(n, m) \mid n \text{ is divisible by } m, m \neq 1, m \neq n\}$
3. $R = \{(n, m) \mid n^2 + m^2 = 3\}$
4. $R = \{(n, m) \mid n^2 + m^2 = 5\}$

Problem 2 (6 points). Given a set $A = \{a, b, c, e, f\}$, a set $B = \{w, x, y, z\}$, and the relations

$$S_1 = \{(a, x), (b, x), (c, y), (c, z), (f, z)\},$$
$$S_2 = \{(a, y), (b, x), (c, y), (c, w), (e, z), (e, x)\}$$

do the following:

1. draw a graph of each of S_1, S_2,
2. for each of S_1, S_2 find domain, range, and the inverse relation ($S_1^{\sim}$ and $S_2^{\sim}$, respectively),
3. find $S_1 \cap S_2$ and $S_1 \cup S_2$.

Problem 3 (3 points). Consider the following pair of relations:

- $R : Student \times Courses$ stands for "the student takes the course"

  R = { (Ann Smith, CS102), (Ann Smith, French101), (Brian Johnson, Math101), (Brian Johnson, French101), (Carol Brown, Math101), (Carol Brown, Chem200), (Daniel Scott, CS102), (Daniel Scott, Chem200) }

- $S : Courses \times Rooms$ stands for "the course meets in the room". A course that has a lab may meet in two rooms: the classroom and the lab.

  S = { (Math101, Science200), (CS102, Science200), (CS102, Lab1), (Chem200, Science300), (Chem200, Lab2), (French101, Lang100) }.

1. Draw the graphs of both relations in such a way that makes it easy to compute their composition (see Figure 14.1 on p. 213 for an example).
2. Compute composition $R; S$.
3. What is the meaning of the composition?

Problem 4 (10 points). You are given three sets $A = \{a, b, c\}$, $B = \{1, 2, 3\}$, and $C = \{red, white, blue\}$ and three relations:

- $R : A \times C = \{(a, red), (b, blue), (a, white)\}$,
- $S : B \times C = \{(1, blue), (3, red)\}$,
- $T : A \times A = \{(a, a), (b, c), (c, b)\}$

For each of the following operations on relations please state whether the operation makes sense. If it makes sense, then compute the result. If it doesn't, then explain why.

1. $R; S$
2. $T; R$
3. $R; T$
4. $R^{\sim}; T$
5. $R^{\sim}; T^{\sim}$
6. $R; S^{\sim}$
7. $T; T^{\sim}; R$
8. R^2
9. T^2
10. T^0