

CSci 1302 Assignment 6
Due Wednesday, October 11

Problem 1 (6 points). Exercise 40c, d, f, g, h, i p. 110.

Problem 2 (6 points). Exercises 54, 55, 56 p. 110.

Problem 3 (28 points). Prove the following arguments. The domain for all problems is \mathbb{Z} - the set of all integers.

$$A. \quad 1. \quad \forall x.x \cdot 1 = x$$

$$\therefore \forall x.\exists y.x \cdot y = x$$

$$B. \quad 1. \quad \forall x.(x \neq 1 \wedge x \neq 0) \rightarrow x^2 > x$$

$$2. \quad \exists y.y \neq 1 \wedge y \neq 0$$

$$\therefore \exists z.z^2 > z$$

Hint for problem C: when introducing the existential quantifier, replace only one occurrence of the constant by a variable, but not the other:

$$C. \quad 1. \quad \forall z.isDivisible(z, 1) \wedge isDivisible(z, z)$$

$$\therefore \exists y.isDivisible(y, 33)$$

$$D. \quad 1. \quad \forall x.\forall y.(x > y) \vee (y > x) \vee (x = y)$$

$$2. \quad \sim (5 > 5)$$

$$\therefore 5 = 5$$

$$E. \quad 1. \quad \forall x.\forall y.\exists z.x + y = z$$

$$\therefore \forall x.\exists z.x + x = z$$

$$F. \quad 1. \quad \forall x.isPrime(x) \leftrightarrow (\forall y.isDivisible(x, y) \rightarrow (y = 1 \vee y = x))$$

$$2. \quad isDivisible(9, 3)$$

$$3. \quad 3 \neq 1 \wedge 3 \neq 9$$

$$\therefore \sim isPrime(9)$$

$$G. \quad 1. \quad \forall x.odd(x) \leftrightarrow (\sim \exists y.x = 2 \cdot y)$$

$$4 = 2 \cdot 2$$

$$\therefore \sim odd(4)$$