CSci 1302 Assignment 12
Due Friday, Dec. 8

Problem 1 (6 points). You are given three sets, \( A = \{a, b, c\} \), \( B = \{1, 2, 3\} \), and \( C = \{\text{red, white, blue}\} \), and the following relations:

- \( R \subseteq A \times B = \{(a, 1), (b, 1), (c, 3)\} \),
- \( S \subseteq B \times C = \{(2, \text{red}), (2, \text{blue}), (3, \text{red})\} \),
- \( T \subseteq C \times A = \{(\text{red}, a), (\text{white}, c)\} \).

For each of the following operations compute the result if the operation makes sense, or, if it doesn’t make sense, please explain why.

1. \( R; S \)
2. \( S; T \)
3. \( R; R^{-1} \)
4. \( R^{-1}; R \)
5. \( R^{-1}; T^{-1} \)
6. \( R; T \)


Problem 3 (6 points). For each relation on natural numbers \( N = \{1, 2, 3, \ldots\} \) defined below please answer the following questions:

- Is the relation reflexive?
- Is the relation symmetric?
- Is the relation antisymmetric?
- Is the relation transitive?

Please explain the negative answers briefly.

1. \( R = \{(n, m) \mid n + m \text{ is even}\} \)
2. \( R = \{(n, m) \mid n + m \text{ is odd}\} \)
3. \( R = \{(n, m) \mid n \text{ is even}, m \text{ is even}\} \)
Problem 4 (5 points) You are given relation $R = \{(a, b), (b, c), (c, b), (d, c)\}$ on the universal set $U = \{a, b, c, d, e\}$. Please construct the following:

- the reflexive closure of $R$.
- the symmetric closure of $R$.
- the transitive closure of $R$.
- the “equivalence closure” of $R$ (i.e. the smallest equivalence relation that contains $R$).

You may list pairs included in the resulting relations or draw them (each on a separate diagram).

Problem 5 (2 points). Is symmetric closure of a transitive relation transitive? If yes, please prove it. If not, please give a counterexample.