CSci 1302 Assignment 4
Due Wedn., October 5th in class

Problem 1 (5 points). Exercises 15, 17, and 19 p. 55.

Problem 2 (6 points). Exercises 27, 29 p. 56.

Problem 3 (4 points). Exercises 33a, 34a pp. 56-57.

Problem 4 (4 points). Exercise 21 b,c p. 73.

Problem 5 (2 points). Exercises 24, 26 p. 73.

For the next problems assume the domain is the set $\mathbb{Z}$ of all integers: positive, negative, and zero. $x < y$, $x \leq y$, etc. are binary predicates with the obvious meaning. We also use predicates $isOdd(x)$, $isEven(x)$, and $isDivisibleBy(x, y)$, which means that x is divisible by y.

Problem 6 (9 points). Translate the following formuals to English, indicate whether each one is true or false, and briefly justify your answer.

1. $^\sim \forall x.(x^2 > 0) \vee (x = 1)$

2. $\exists x. x^2 \leq x$

3. $\forall x. isOdd(x) \rightarrow isEven(x)$

4. $\exists x.(x \leq 2) \rightarrow (isOdd(x) \wedge isEven(x))$

5. $(\exists x.(isOdd(x) \wedge isEven(x))) \rightarrow (\forall x.(isOdd(x) \wedge isEven(x)))$ (you don't need to translate this one to English, just say if it's true or false and explian why).

6. $\forall x. x \leq x$

7. $\forall x. \exists y. isDivisible(x, y)$

8. $\exists x. \forall y. isDivisible(y, x)$

9. $\forall x. \exists y. isOdd(x) \rightarrow isOdd(y)$

Problem 7 (10 points). Write the following sentences as quantified formulas. Note that some of these formulas need more than one quantifier.

1. Every number is divisible by 1.

2. Some numbers are divisible by 3.

3. Not all numbers are divisible by 3.

4. No odd number is divisible by 2.

5. No number is greater than itself.

6. Squares of odd numbers are odd.

7. No squares of even numbers are prime.

8. No matter what pair of numbers you take, either the first one is greater than the second one, or the second one is greater than the the first one. (**Extra credit:** is this a true statement? Justify your answer.)

9. Every number is divisible by some number.

10. Some numbers are squares of some other numbers (don't use the predicate $isSquare(x)$).