CSci 1302 Assignment 7

Due Fri., Nov. 5th, 2004

This problem set uses some of the following predicates on integer numbers:

- Unary: $prime(x), even(x), odd(x)$ mean "x is prime", "x is even", "x is odd", respectively. $thelargest(x)$ means that x is the largest of all numbers.
- Binary: $equal(x, y)$ means "x is equal to y", $greater(x, y)$ means "$x > y$", $divisible(x, y)$ means "x is divisible by y".
- Ternary: $sum(x, y, z)$ means that $x = y + z$.

While the meaning of the predicates is not important for the proofs, it might be helpful in providing intuition about the statements.

Use subscripts (such as $x_\exists$ and $x_\forall$) to distinguish between unknown and genuine variables after instantiation and specify the rules as we did in class.

Problem 1 (9 points). Prove the following arguments in the predicate logic.

1. $\forall x.\forall y.sum(x, x, y) \Rightarrow equal(y, 0)$
   $\exists z.sum(5, 5, z)$
   ————————————
   $\exists z.equal(z, 0)$
2. $\forall x.even(x) \Leftrightarrow divisible(x, 2)$
   $\exists x.even(x)$
   ——————
   $\exists x.\exists y.divisible(x, y)$
3. $\forall x.\forall y.greater(x, y) \lor greater(y, x)$
   $\forall x.\neg greater(x, x)$
   ——————
   false

Problem 2 (8 points). Exercise 9.4 p. 131, parts 1, 2 only.

Problem 3 (9 points). Prove the following arguments in the predicate logic:

1. $\forall x.divisible(x,1)$
   $\neg\forall x.equal(x,1)$
   ———————————————
   $\exists z.\exists w.divisible(z,w) \wedge \neg equal(z,w)$

2. $\forall x.\exists y.sum(y,x,x)$
   $\forall z.\forall y.sum(z,y,y) \Rightarrow even(z)$
   ———————————————
   $\exists w.even(w)$

3. $\forall x.\forall y.greater(x,y) \Rightarrow \exists z.sum(x,y,z)$
   $\forall x.\exists y.greater(x,y)$
   ———————————————
   $\forall x.\exists y.\exists z.sum(x,y,z)$